AMIRKABIR WINTER SCHOOL

Minimalism in Robotics:

From Sensing to Filtering to Planning

PART 5: PLANNING IN INFORMATION SPACES

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March 5, 2012

Overview of Topics

From filters to planning
General issues
Visibility-based pursuit evasion
Maze searching
Gap navigation trees
Learning convex hulls of landmarks
Bug algorithms
Sensorless manipulation
Controlling Wild Bodies

1. An I-space view of planning, starting with temporal filters
2. General planning issues
3. Maze searching
4. Visibilty-based pursuit-evasion
5. Shadow information spaces
6. Gap navigation trees
7. Landmark-based navigation
8. Bug algorithms
9. Sensorless manipulation
10. Wild bodies

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From filters to planning

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Let $\mathcal{I}$ be any I-space.

Assume a filter

$$\iota_k = \phi(\iota_{k-1}, u_{k-1}, y_k)$$

is given.

Let $G \subset \mathcal{I}$ be a *goal region*.

Starting from ι_0, what sequence of actions u_1, u_2, . . ., will lead to some future I-state $\iota_k \in G$?

Using Filters For Planning

From filters to planning

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The future may be unpredictable.

Introduce an *I-state dependent* plan:

$$\pi : \mathcal{I} \rightarrow U$$

Using a filter ϕ, the execution of a plan can be expressed as

$$\iota_k = \phi(\iota_{k-1}, y_k, \pi(\iota_{k-1}))$$

The I-space $\mathcal{I}$ is just a sort of “C-space” that is being explored.

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General issues

Main Issues

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The following issues arise repreatedly in planning:

1. **Predictability**
2. **Reachability**
3. **Optimality**
4. **Computability**

Predictability

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Are the effects of actions predictable in the I-space $\mathcal{I}$?

If **yes**, then a *path* through the I-space is obtained.

Example: Sensorless manipulation

Example: Visibility-based pursuit evasion

By analogy to path planning in C-space:

1. Combinatorial planning in I-space
2. Sampling-based planing in I-space

If **no**, then information feedback is critical

It is like feedback planning (or control) in C-space, but instead over I-space

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Reachability:
Is the goal region $G \subset \mathcal{I}$ even reachable from the initial I-state?

Do there even exist actions that will take us to G?

Does there exist a plan that can reach G?

With unpredictability, is G *guaranteed* to be reached, over all possible disturbances?

A more basic question is whether the goal can even be adequately expressed in $\mathcal{I}$.

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Perhaps many plans can reach G

What criteria should be formulated to compare plans?

Which plans are the best, or optimal with respect to criteria?

Do optimal plans even exist?

Computability

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Given a description of the problem, can an algorithm be determined that automatically computes a useful plan?

Sometimes a clever human designs the plan (e.g. bug algorithms)

What is the algorithmic complexity of computing a solution plan?

What is the implementation difficulty of computing a solution plan?

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Important Generic Examples

State feedback: I-space is $\mathcal{I} = X$ and plan is $\pi : X \rightarrow U$

Open loop: $\mathcal{I} = \mathbb{N}$ and $\pi : \mathbb{N} \rightarrow U$
π can be written as $(u_1, u_2, u_3, \ldots)$

Sensor feedback: $\mathcal{I} = Y$ and $\pi : Y \rightarrow U$

History feedback $\mathcal{I} = \mathcal{I}_{hist}$ and $\pi : \mathcal{I}_{hist} \rightarrow U$

Recall the previous filters over these 4 I-spaces.

Now we move from *passive* to *active*.

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Overall Process

Based on the task, an overall approach that leads to planning:

1. Design the system, which includes the environment, bodies, and sensors.

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Overall Process

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1. Design the system, which includes the environment, bodies, and sensors.
2. Define the models, which provide the state space X, the sensor mapping h, and the state transition function f.

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1. Design the system, which includes the environment, bodies, and sensors.
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3. Select an I-space $\mathcal{I}$ for which a filter ϕ can be practically computed.

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4. Take the desired goal, expressed over X, and convert it into an expression over $\mathcal{I}$.

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3. Select an I-space $\mathcal{I}$ for which a filter ϕ can be practically computed.
4. Take the desired goal, expressed over X, and convert it into an expression over $\mathcal{I}$.
5. Compute a plan π over $\mathcal{I}$ that achieves the goal in terms of $\mathcal{I}$.

Really, all steps should be considered together.

Might have to backtrack.

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Visibility-based pursuit evasion

Model

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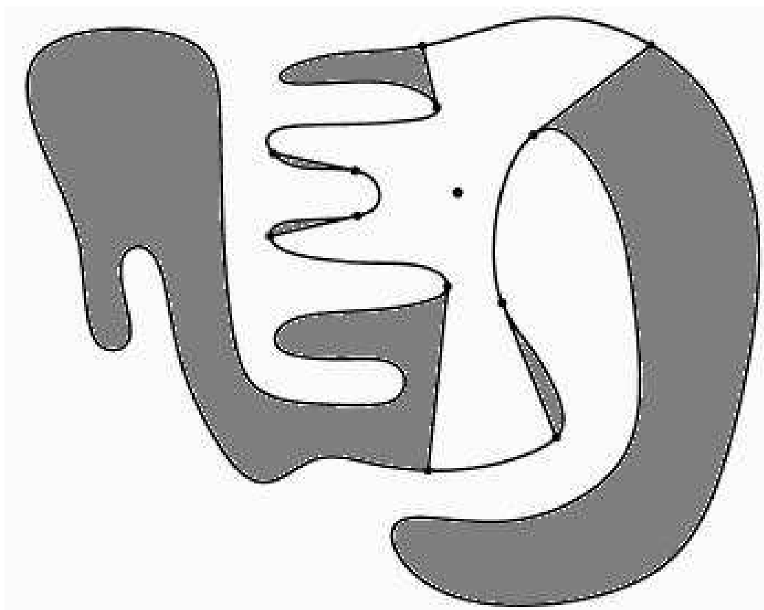
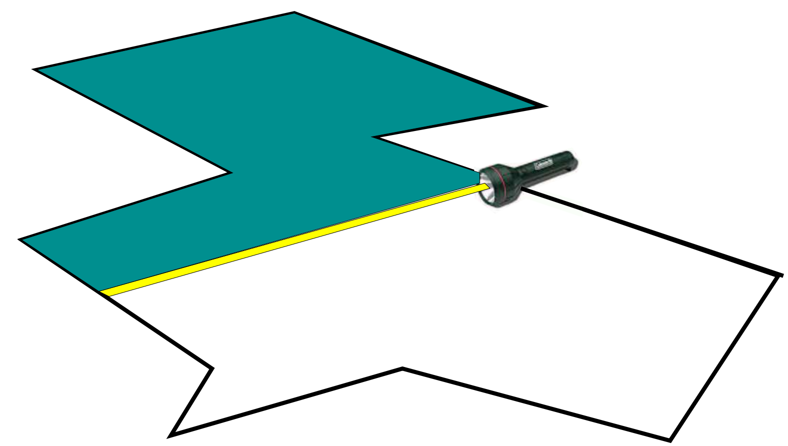
- A 2D environment, possibly curved
- Unpredictable point “evaders” move with unbounded speed
- Point “pursuers” use visibility sensors to find all evaders





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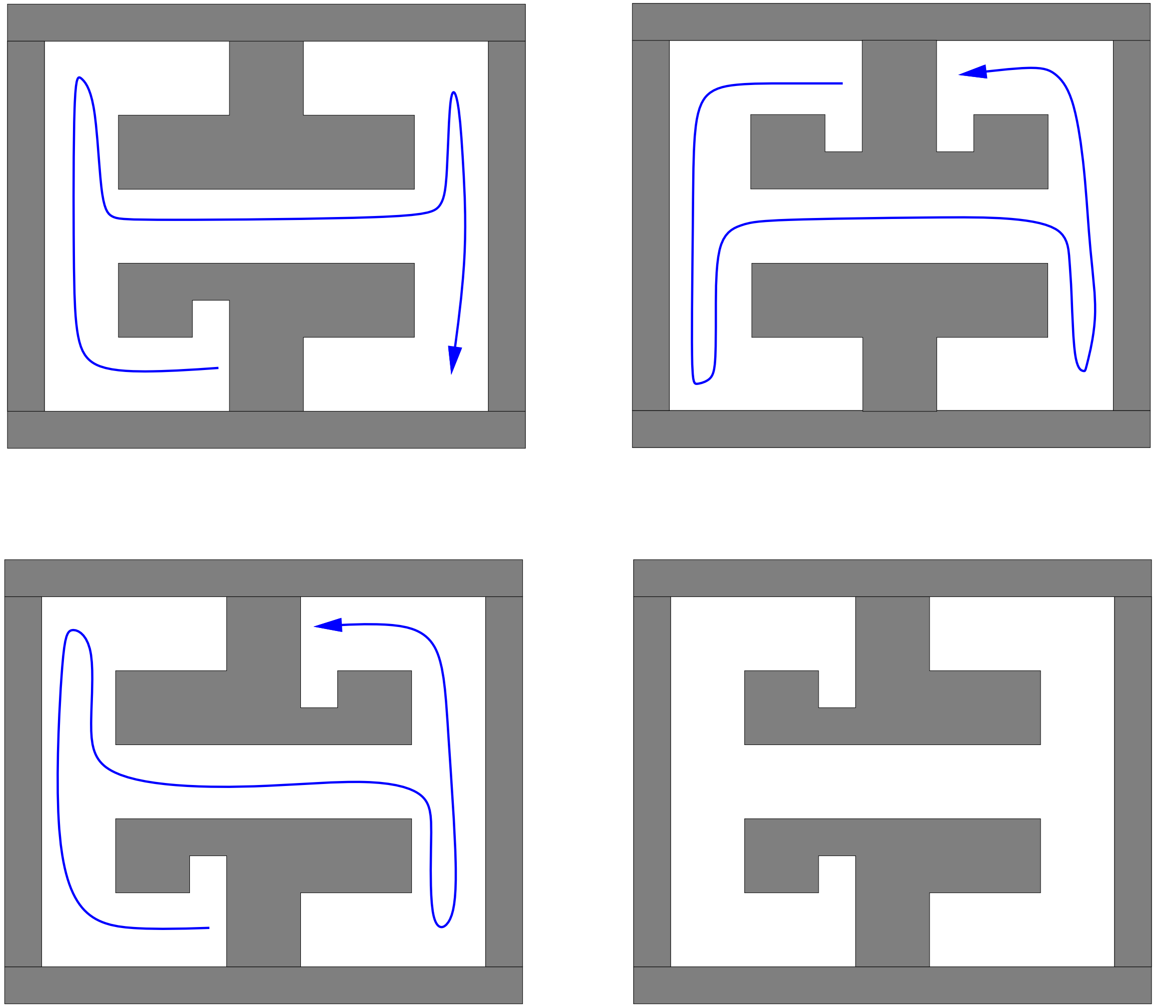
Try to solve using one pursuer with 360° vision:



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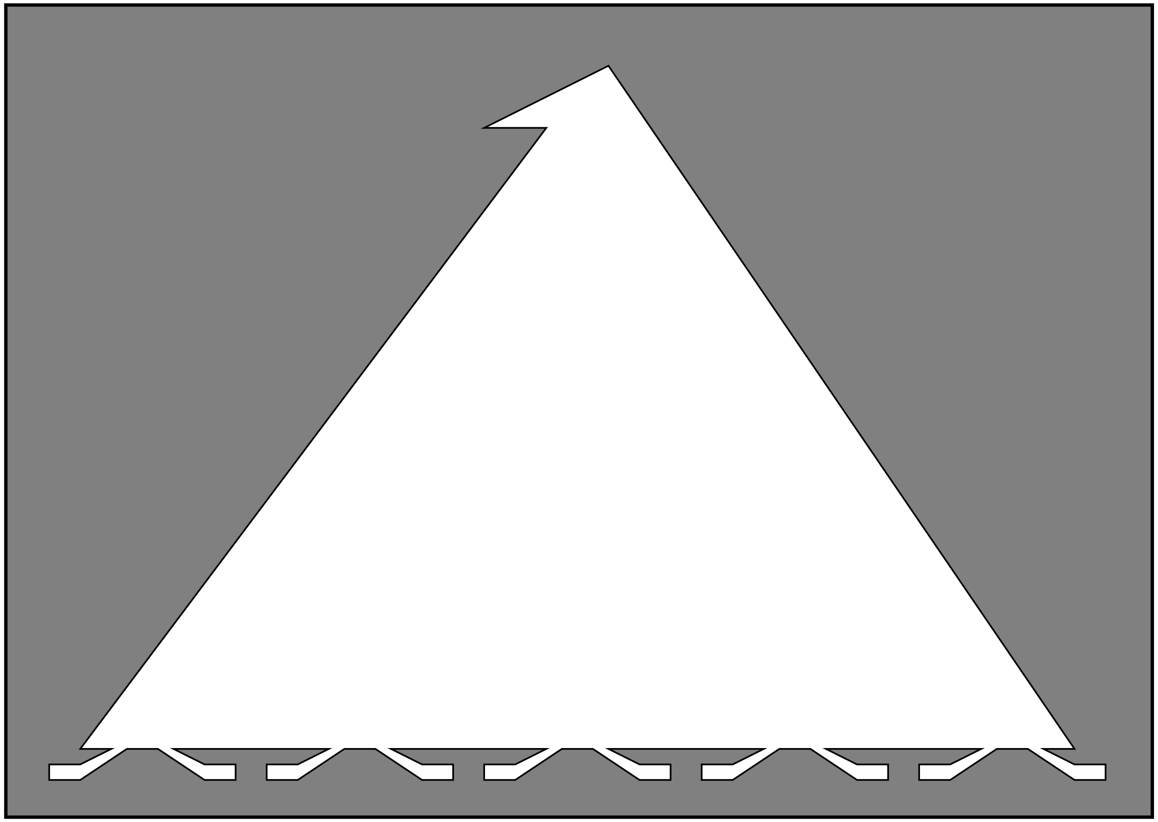
You might have to revisit the same place many times...



$\Omega(n)$ recontaminations

A Cell Decomposition

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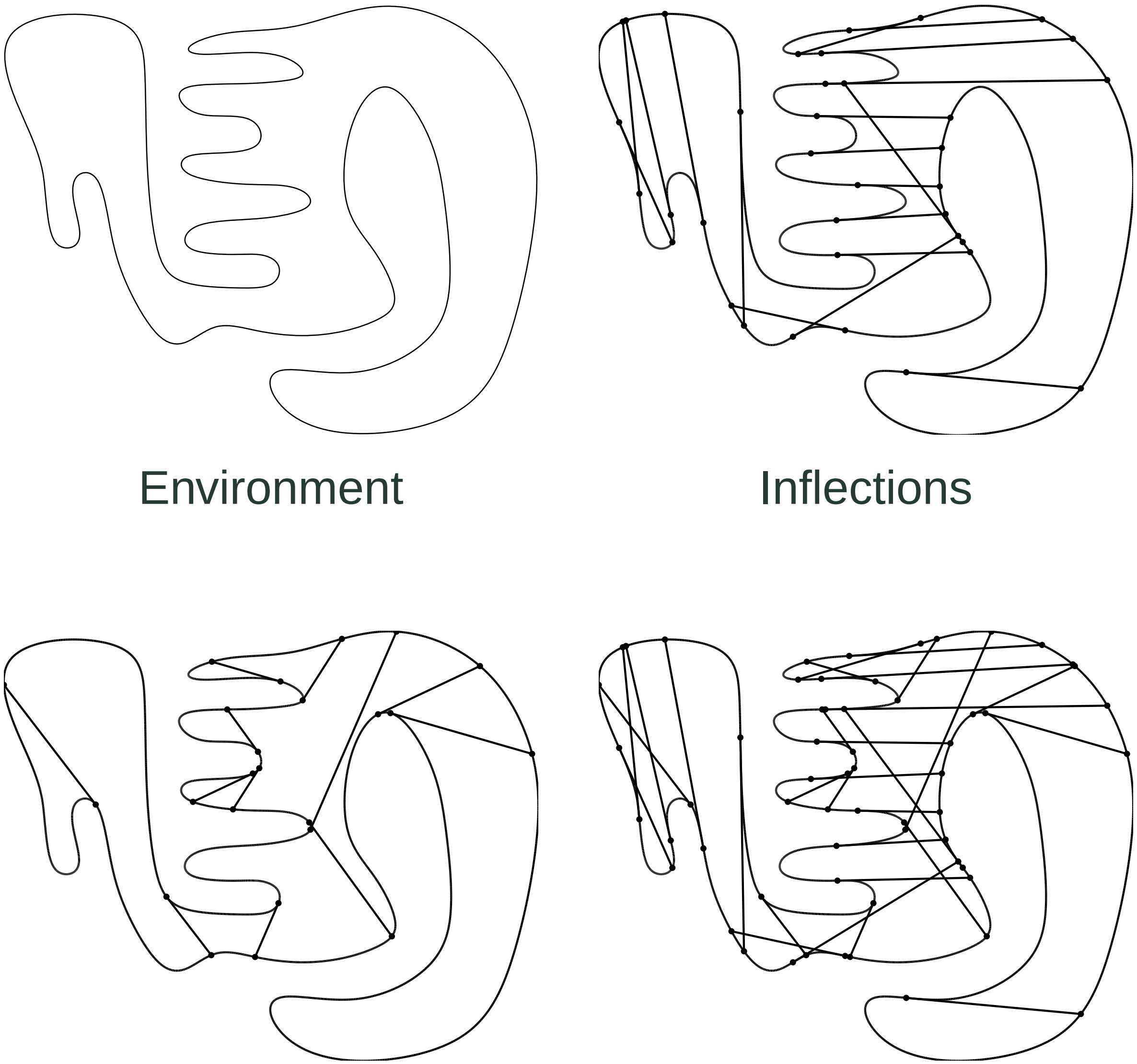


Environment

Inflections

Bitangents

Cell Decomposition

The Information Space

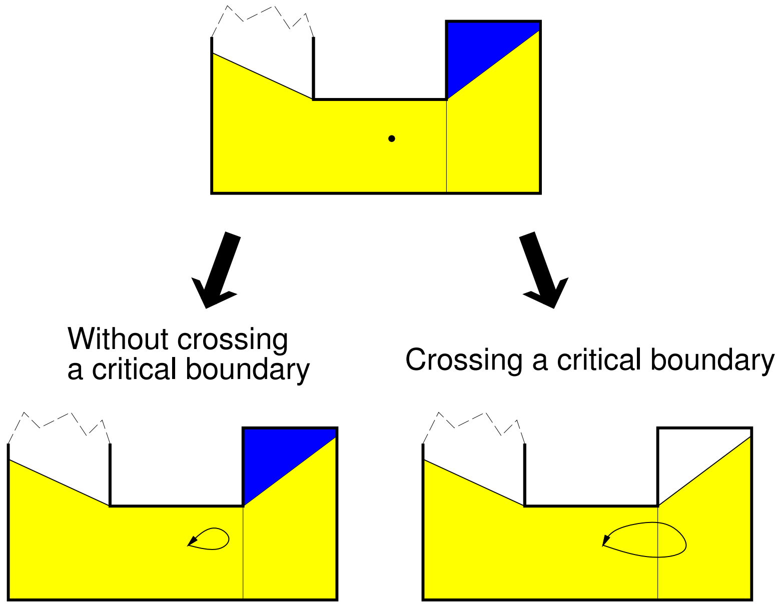
Identify all unique situations that can occur:

An information state is identified by (x, S) in which

x = *the position of the pursuer*

S = *set of possible evader positions*

The set of all information states forms an information space.



Many closed-path motions retain the same information state.

Systematic Graph Search

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- Let $G(V, E)$ be the dual of the cell decomposition
- For each $v \in V$, there are finitely many information classes
- Form a directed information state graph, $G_I(V_I, E_I)$
- Each $v \in V_I$ is an information class
- Each $e \in E_I$ indicates a transition between information classes (crossing an inflection or bitangent)

For each information class, label each shadow component with “1” for *contaminated* or “0” for *clear*.

Search G_I from a state in which

All labels are “1”

to a state in which

All labels are “0”

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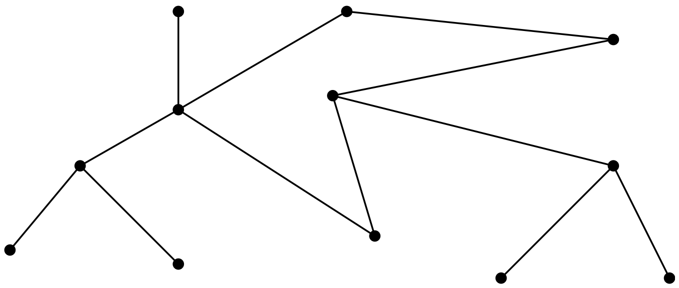
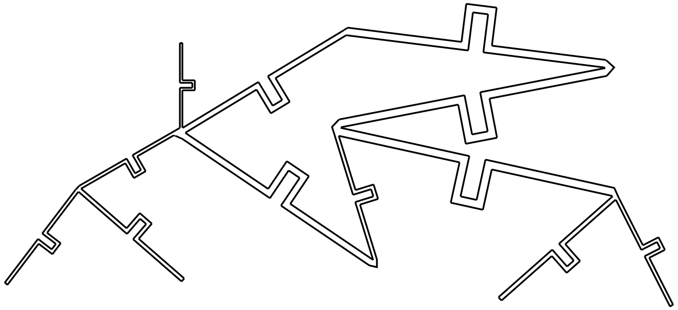
Pursuit-evasion planar graph problem:



A geometric equivalent:



Results:

Deciding whether a simple polygon can be searched by k pursuers is NP hard.

$\Omega(\lg n + \sqrt{h})$ pursuers needed for some polygons

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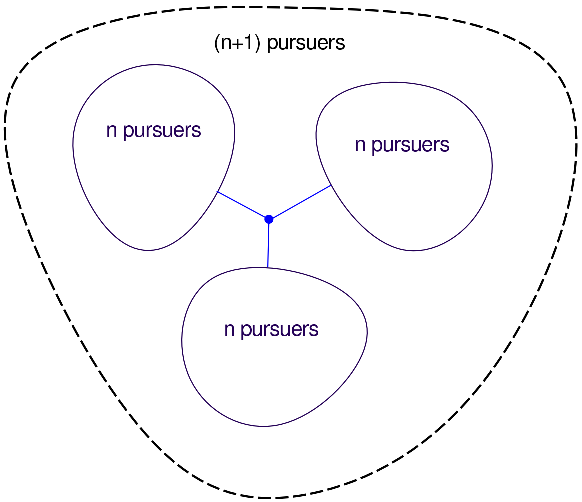

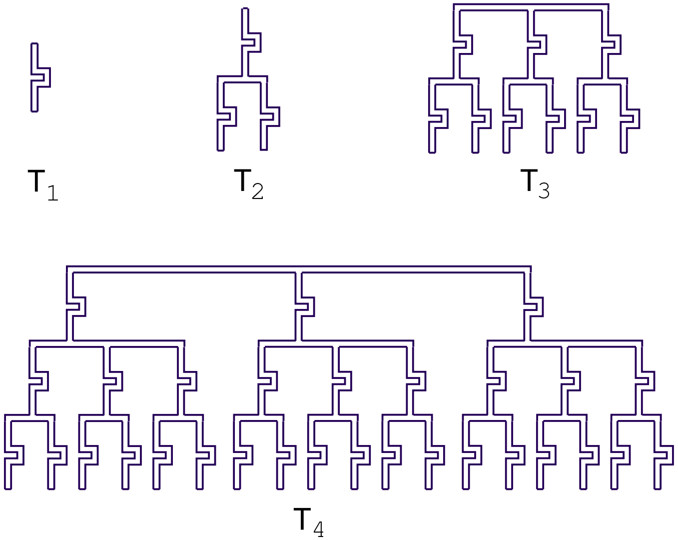

This sequence requires $\Omega(\lg n)$ pursuers.

Pursuit-Evasion Results with Perfect Models

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- Constructing and searching equivalence classes in the information space
- A complete algorithm for 360° visibility
- A complete algorithm for 1 pursuer with 1 flashlight
- A complete algorithm for 2 pursuers with 1 flashlight each

All of these assume perfect mapping, control, and localization.

Alternative pursuit-evasion approaches:

- Using the gap sensor (Sachs, Rajko, LaValle, IJRR 2004)
- Using a wall-following robot (Katsev, Tovar, Yershova, Ghrist, LaValle, IEEE Trans. Robotics, 2011

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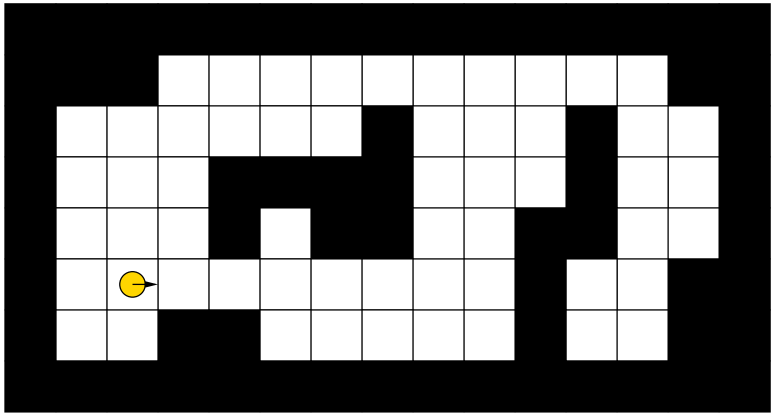

Each $E \in \mathcal{E}$ is a bounded set of white tiles.

$$X \subset \mathbb{Z} \times \mathbb{Z} \times D \times \mathcal{E}$$

Actions: 1) Rotate 90 degrees CCW; 2) Move foward one tile.

Task: Make a plan that systematically searches all white tiles.
For example, find a hidden treasure.

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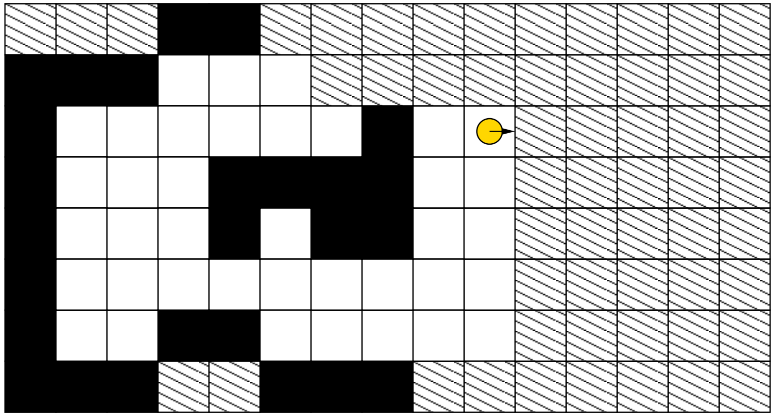
Could try $\mathcal{I} = \mathrm{pow}(\mathbb{Z} \times \mathbb{Z} \times D \times E)$.
Too large!

Instead, maintain I-states B (known black tiles) and W (known white tiles).



All other tiles assumed "unknown".

I-space $\mathcal{I}$ is all ways to partition $\mathbb{Z} \times \mathbb{Z}$ into connected "white", "black", and "unknown" tiles.
Linear space required for an I-state (filter memory).

Maze Searching: Blum and Kozen 1978

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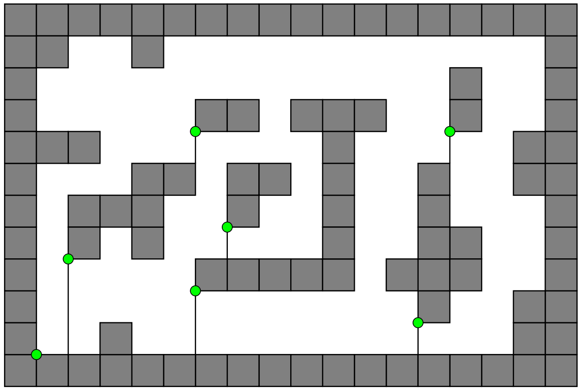

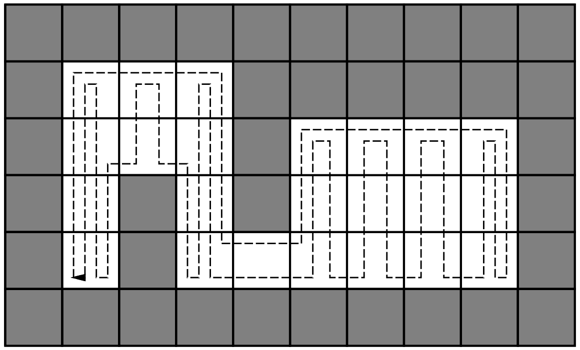

I-state: Latitude (integer) and orientation (two bits)

Only logarithmic space required: Not enough for a “map”.

They found an I-space that is much smaller than the set of all maps.

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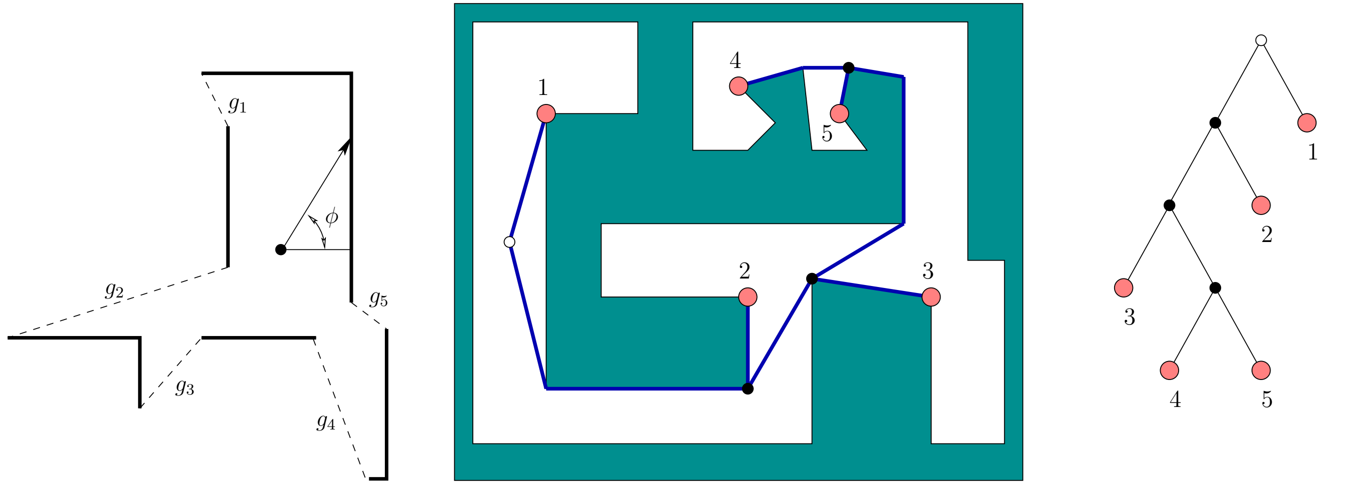
Gap navigation trees

Make Gap Navigation Trees Active



For gap navigation trees, two active tasks:

1. Full exploration of the environment
2. Distance-optimal navigation to retrieve objects

Tovar, Murrieta, LaValle, *IEEE Trans. Robotics*, 2007.

Introduce an Action

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A gap-chasing action is introduced:

Move the robot toward a gap g until a critical event occurs.

One of two events must occur:

1. The gap g splits into two gaps g' and g''.
2. The gap g disappears.

Full Exploration

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If a gap ever appears, mark it as *primitive*.

This is an extension to the filter I-state.

1. Mark all gaps in the initial tree as *non-primitive*.
2. Let $k = 1$.
3. Chase any gap g that is a non-primtive leaf.
4. If g *disappears*, then go to Step 6.
5. If g *splits*, then chase one of its children.
6. Unless all leaves are primitive, increment k and go to Step 3.

At the end, all leaves are primitive and the environment has been fully explored.

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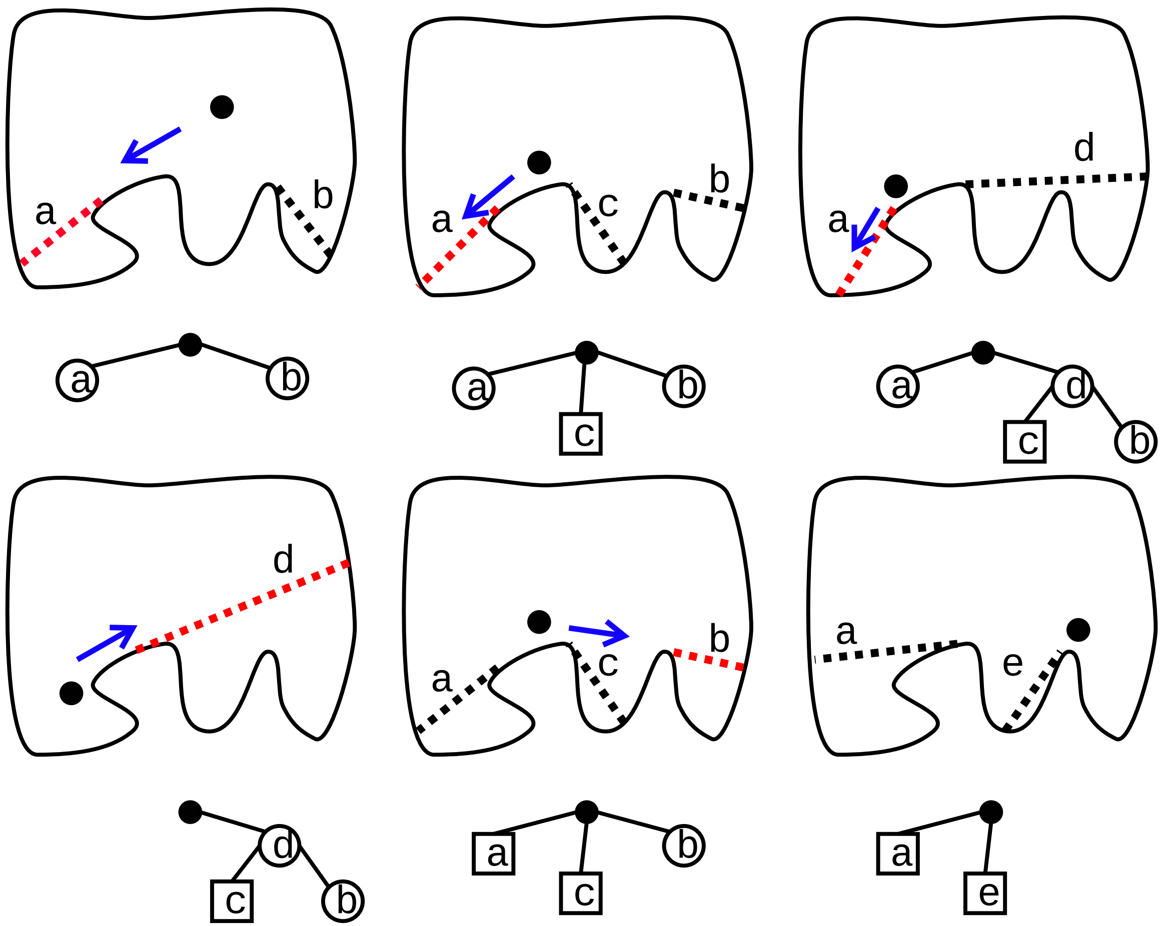
Chase every non-primitive leaf:



Eventually, all leaves become primitive.

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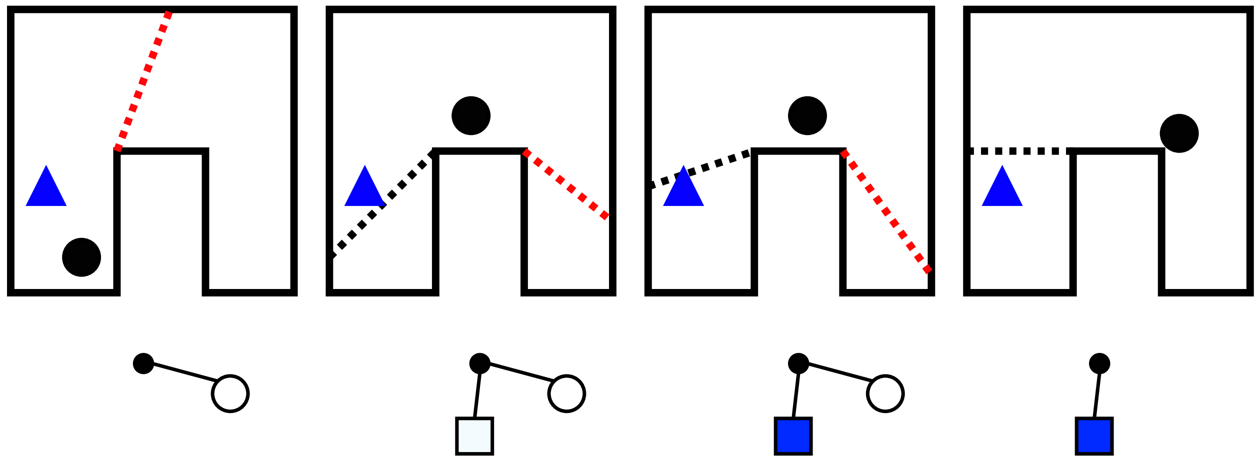
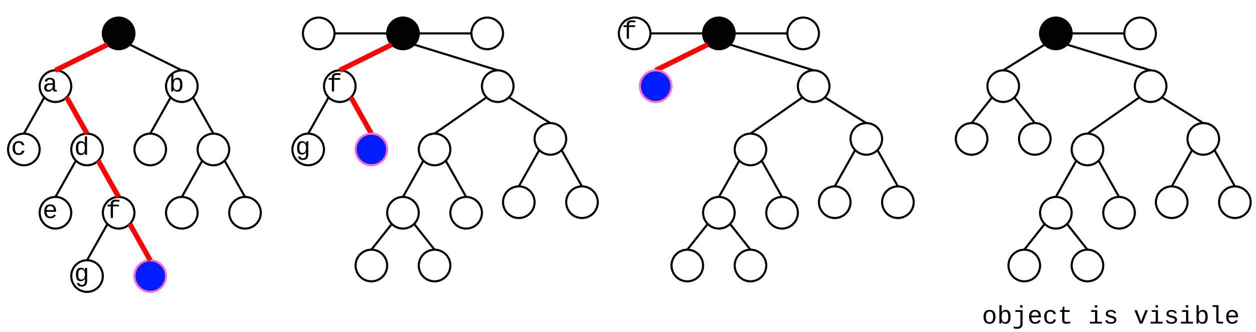
There are no coordinates.



Objects hide behind gaps.



Chase the appropriate sequence of gaps.

Possible Current States

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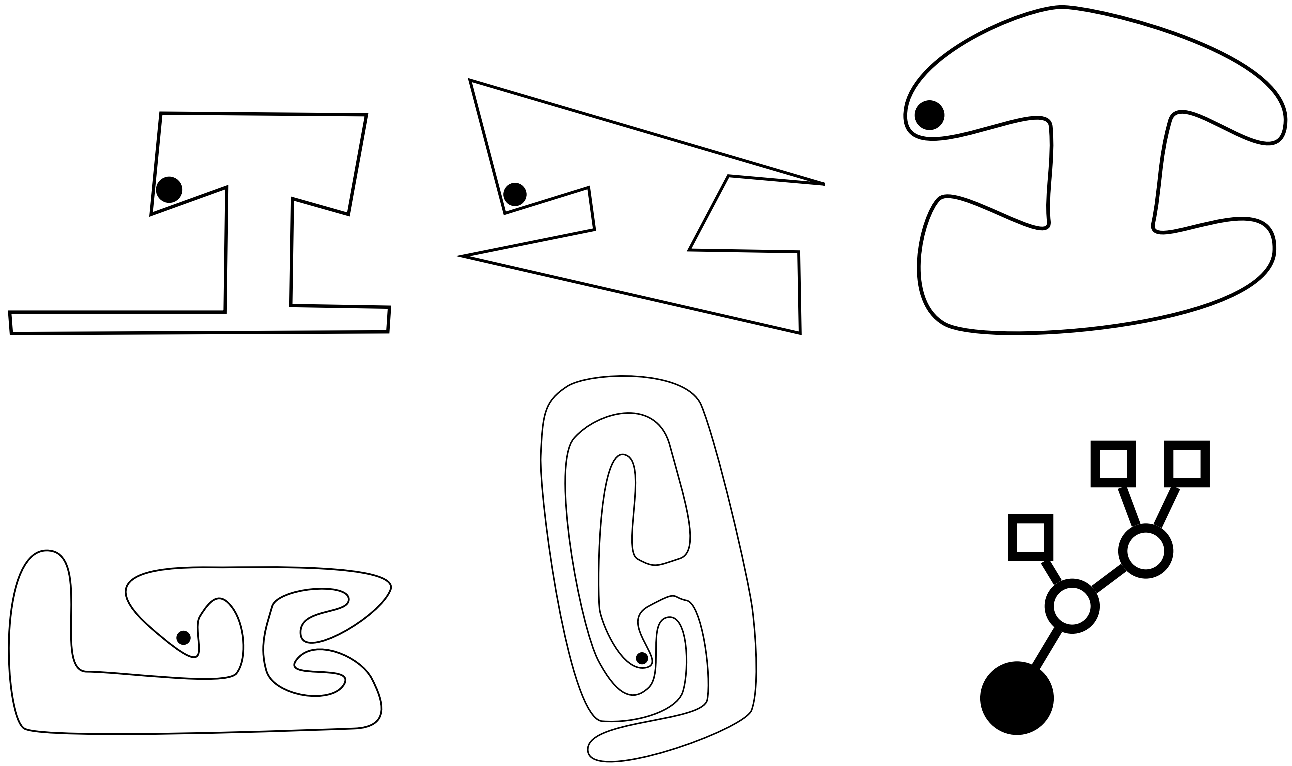

Many configuraton-environment pairs have the same tree.

The robot does not have to distinguish!

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Learning convex hulls of landmarks

Recall: Cyclic Permutation Sensor

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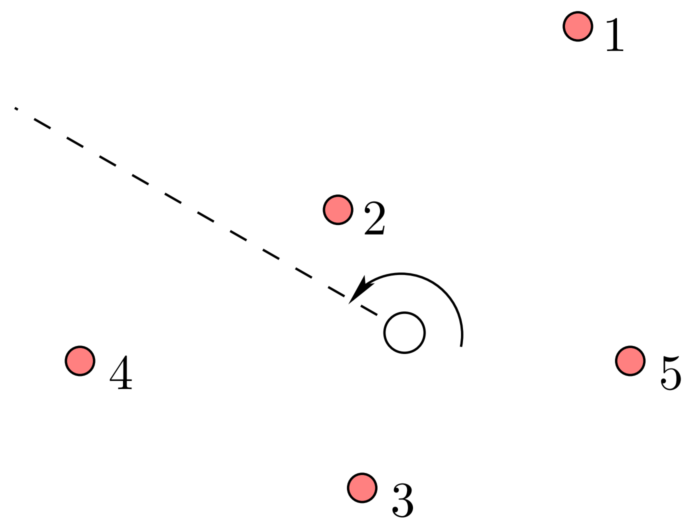
Relation: “is to the left of in counterclockwise order”



Observation: $y = (1, 2, 4, 3, 5)$

Making an Active Version

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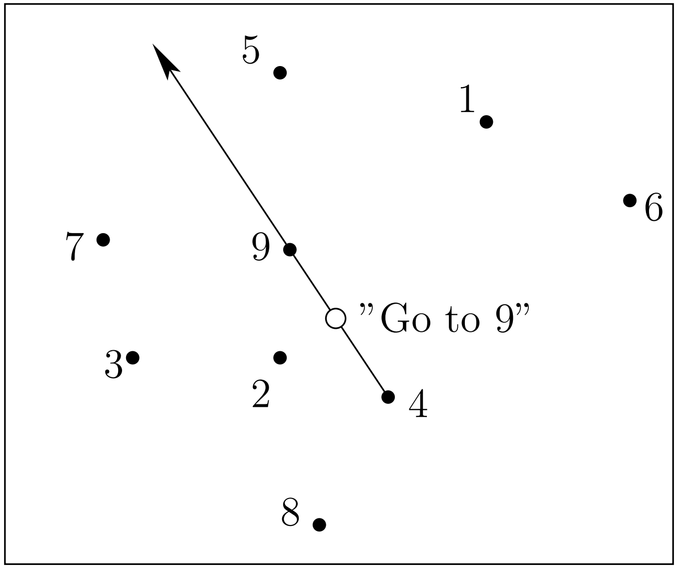
Tovar, Freda, LaValle, 2007.

- Landmark locations are unknown
- Introduce action: “Go to landmark i”
- Can notice which landmarks are “to the left” of the path.



Sense that $(6, 1, 5)$ is to the right of $(7, 2, 3, 8)$.

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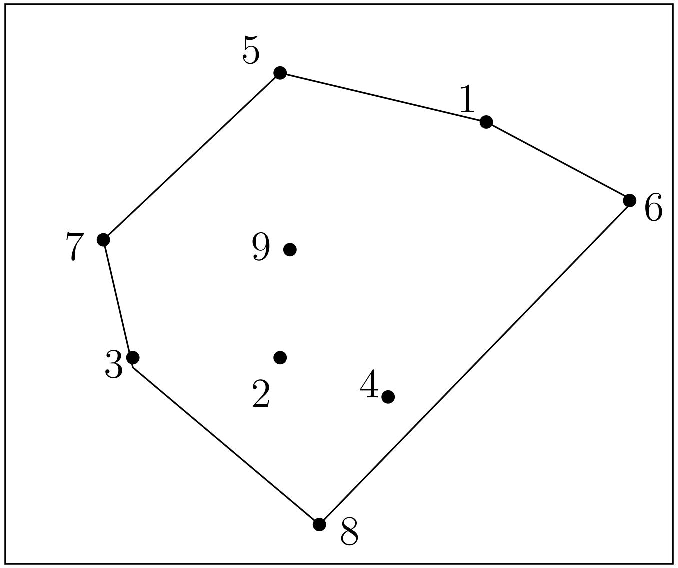

By visiting all pairs, the filter can learn:

- For any subset $L' \subset L$ of landmarks, which others in L lie in the convex hull of L'.
- Equivalently, the robot learns the dual arrangement, order types, oriented matroid.
- The robot can navigation to any goal specified as a cyclic permutation.

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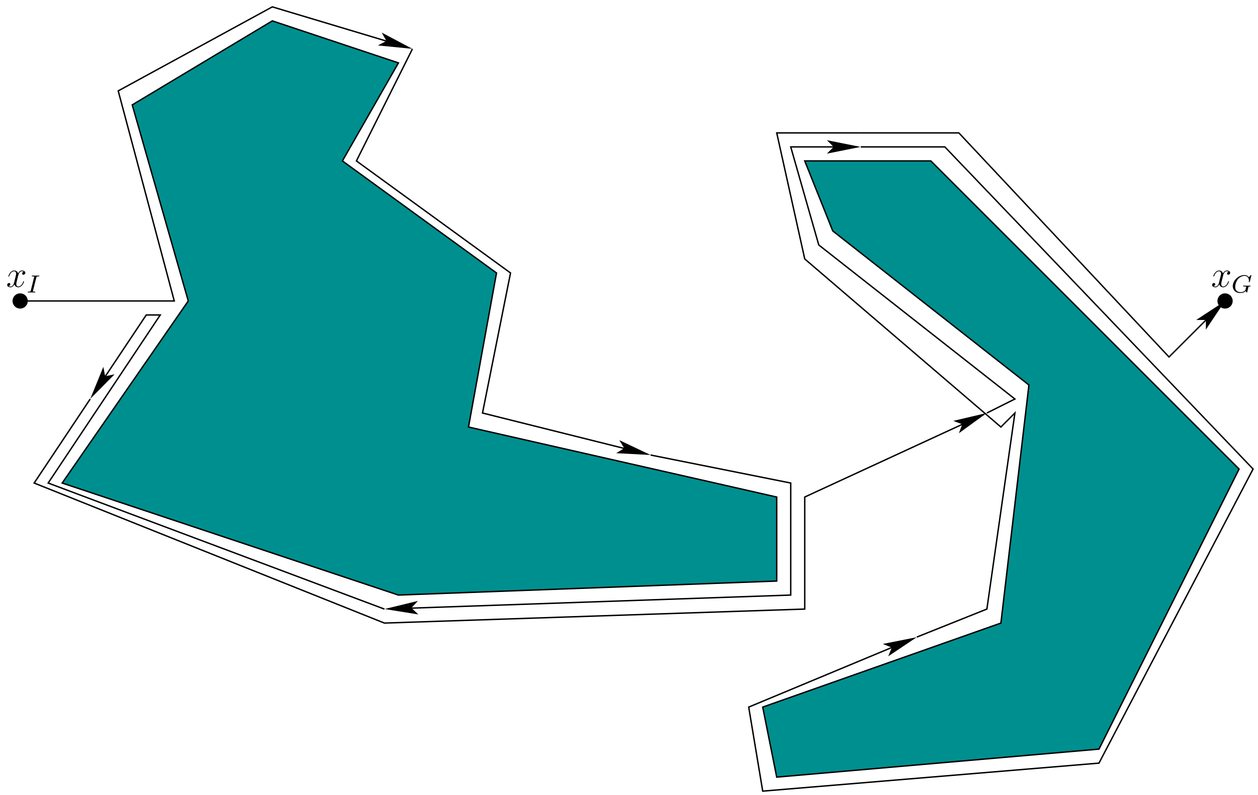

Navigate without being given an initial map E

Lumelsky, Stepanov, 1987; Kamon, Rivlin, Rimon, 1997; many others...

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Taylor, LaValle, ICRA 2009

- The plane contains unknown obstacles with piecewise analytic boundary.
- Each obstacle boundary has finite length.
- A *tower* sends a constant signal.
- Robot has very limited sensors.
- Command the robot so that it reaches the tower.

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- **Boundary (or Contact) sensor:**
  Indicate whether or not robot is on the boundary.
- **Tower alignment/gradient sensor:**
  Indicate whether robot is facing the tower (or intensity gradient).
- **Transformed signal intensity sensor:**
  Observe the value of $m(p - p_t)$.

Regarding m:

- m has only only local maximum, at the tower.
- The function m itself is not given.
- Level sets of m may be symmetric (circles) or asymmetric.

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Actions

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There are three possible actions:

u_{fwd}: Go straight until either ∂E is hit, tower is hit, or local intensity maximum detected.

u_{fol}: Follow ∂E until local maximum detected.

u_{ori}: Rotate until facing tower (or local gradient).

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Illustration of the Plan

Guaranteed to converge; upper bound on distance shown.

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State space: $X \subset \mathbb{R}^2 \times S^1 \times \mathcal{E}$

I-space: $\mathcal{I} = Y^3 \subset \mathbb{R}^3$

I-state components:

1. Current observation
2. Observation when obstacle was last contacted
3. Observation just prior to application of u_{fwd}

Multiple Iterations in the Interior

From filters to planning
General issues
Visibility-based pursuit evasion
Maze searching
Gap navigation trees
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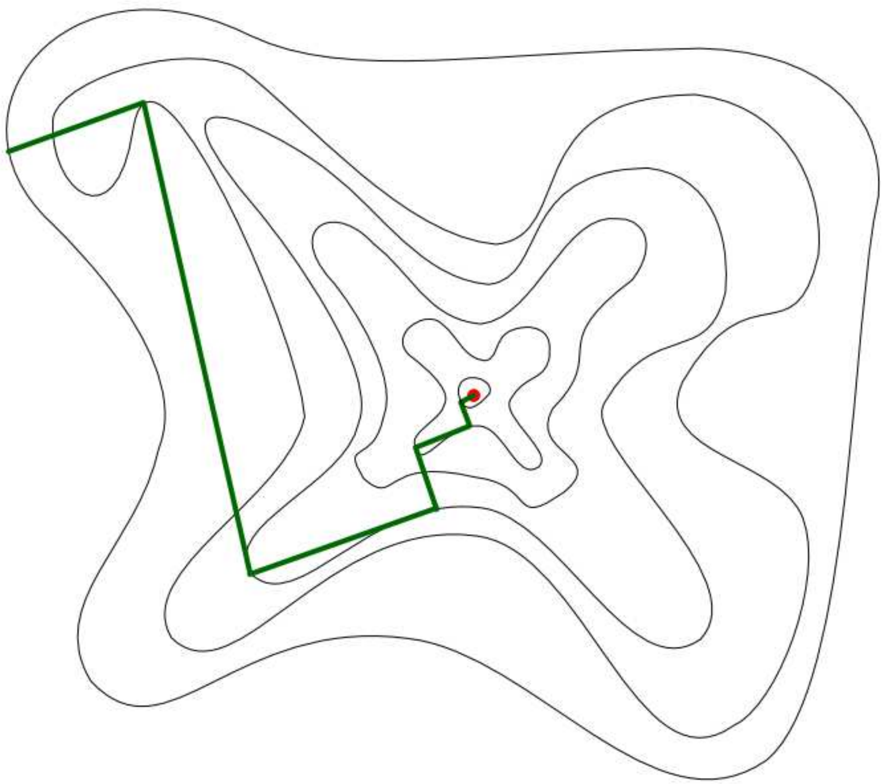

- Equivalent to Steepest Descent with Line Search.
- Result: Convergence is obtained, but distance bound depends on properties of m.

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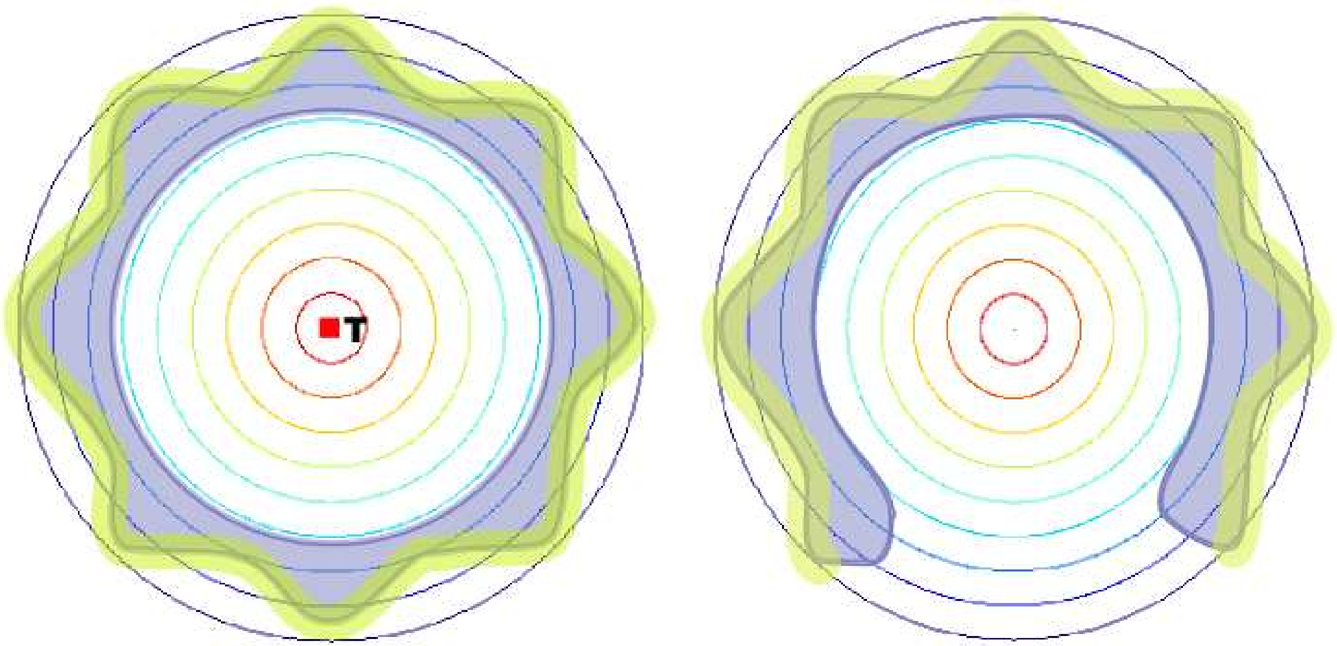
Proposition: Using its sensors and motion primitives, it is impossible for the robot to determine whether the tower is reachable, in other words whether $p_t \in E$.



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Sensorless manipulation

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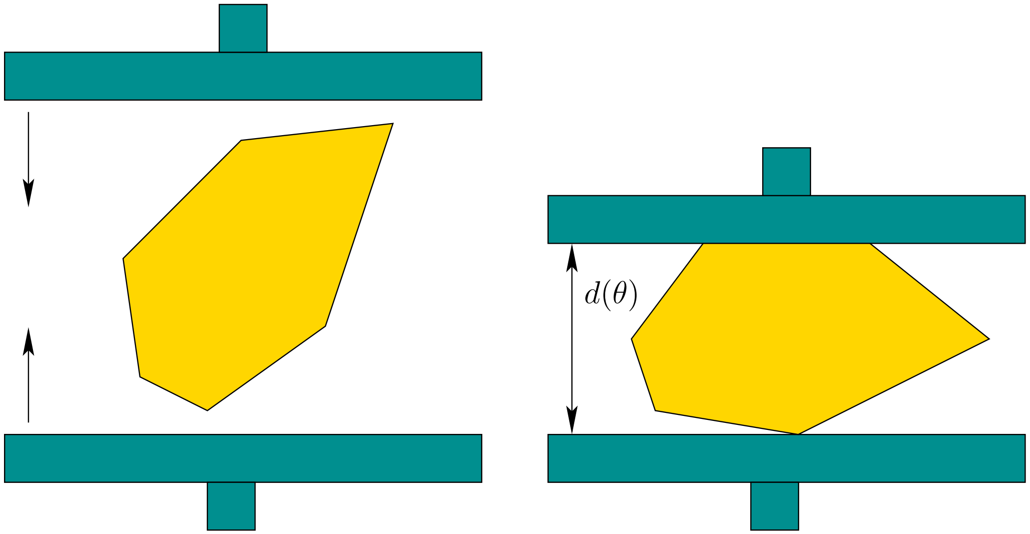
Try to force parts into a known orientation



Mason, Goldberg, 1990

$$X = S^1 \quad \mathcal{I} = \mathrm{pow}(X)$$

Plan: $\pi = (u_1, u_2, \ldots, u_n)$
A sequence of squeeze operations

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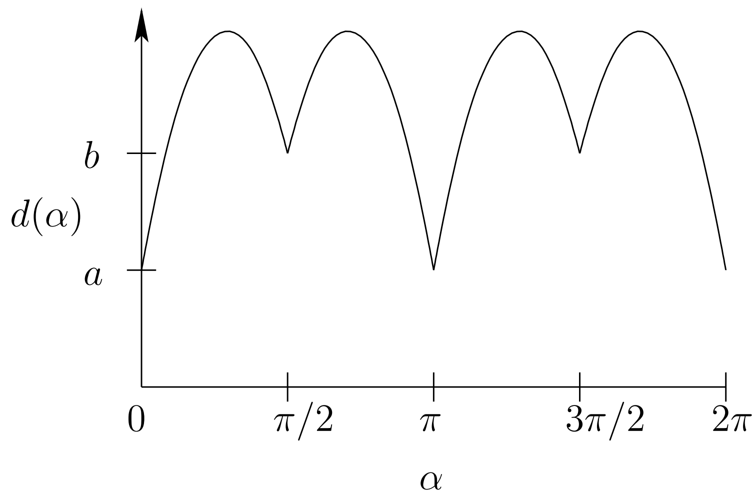

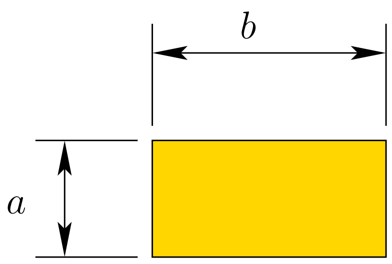

Consider the “diameter” as a function of orientation.

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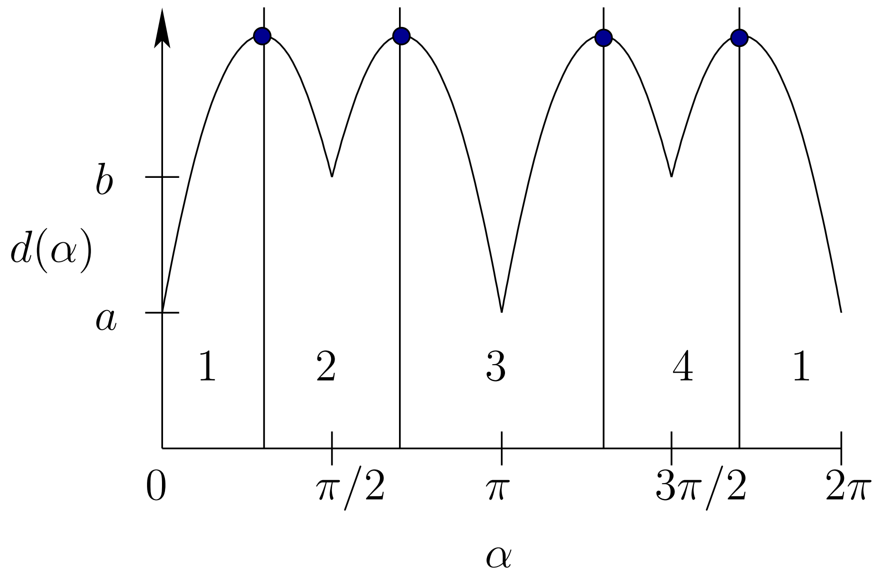

There are four regions of attraction.

This causes a funneling effect when actions are applied.

Sensorless Manipulation

From filters to planning

General issues

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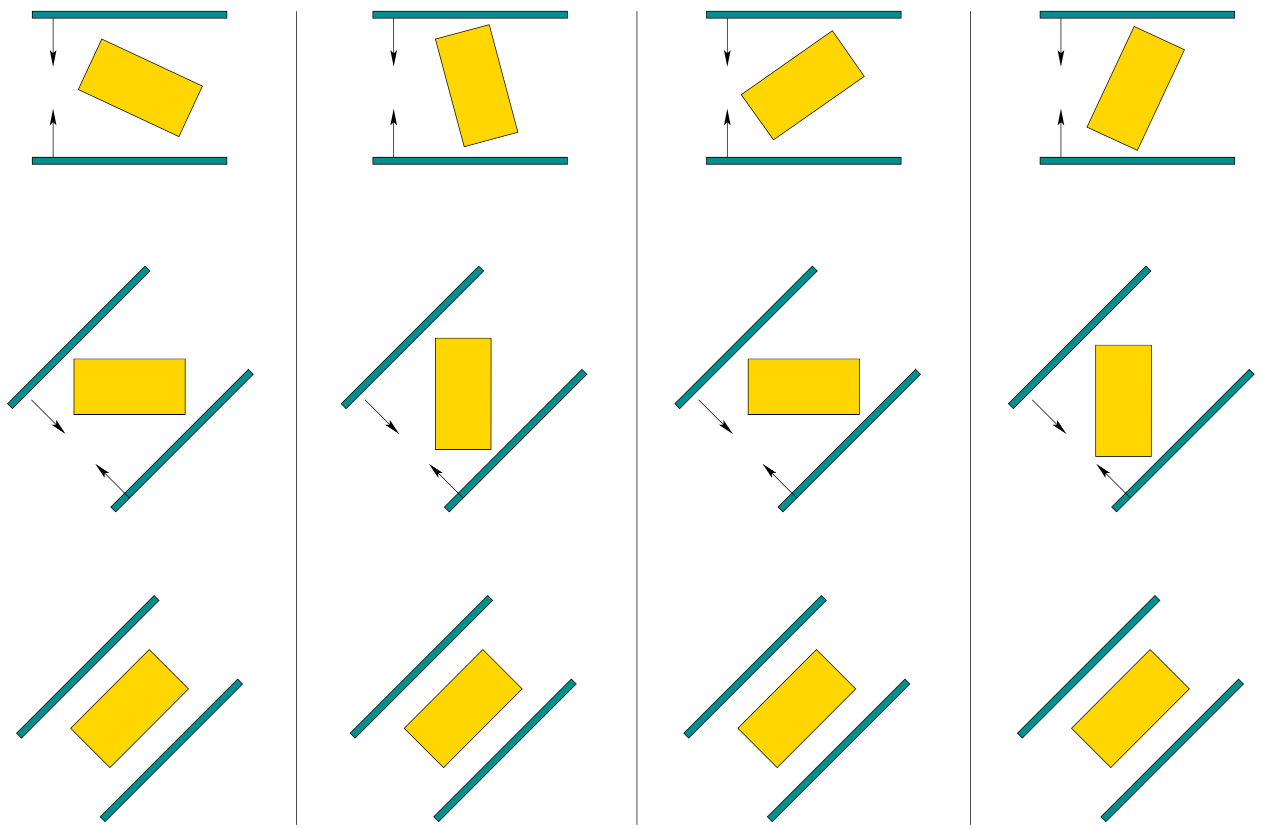
Controlling Wild Bodies



A computed plan that applies two squeeze actions

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Controlling Wild Bodies

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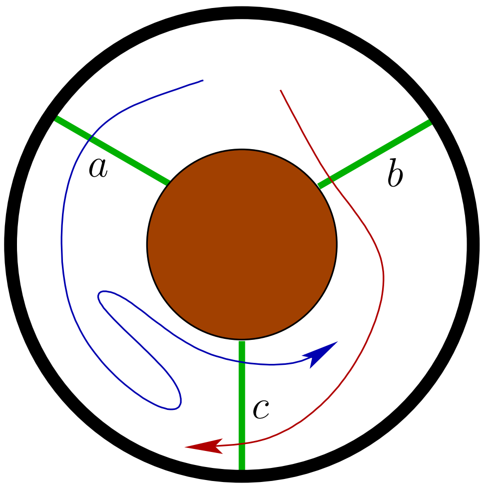
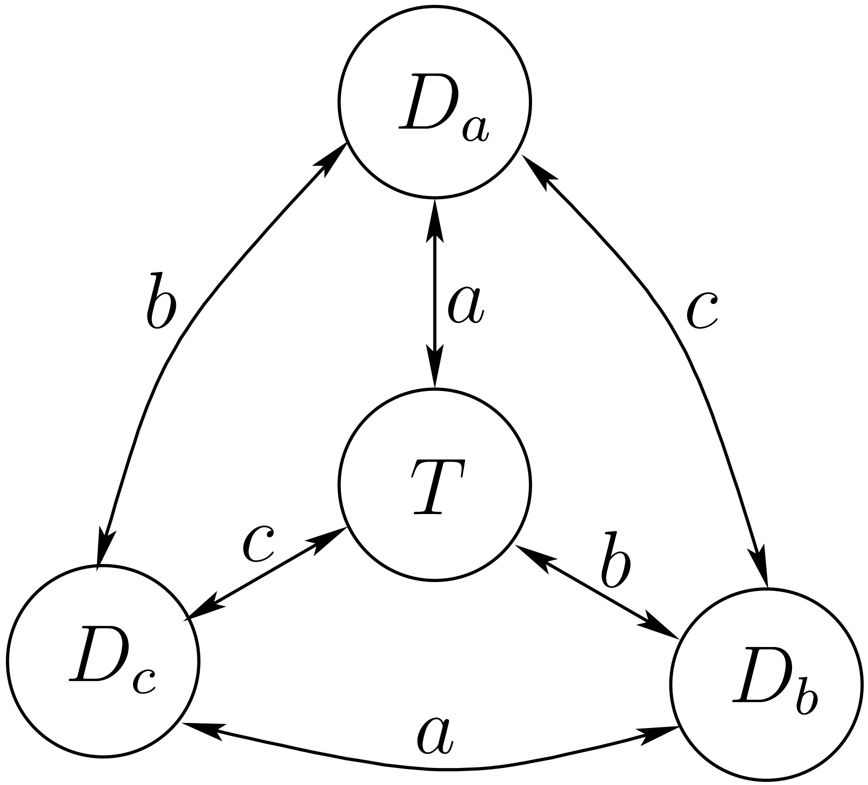
Recall: Two-Bit Filter

Recall the simple filter that determines whether two bodies are in the same region.





Recall: Visibility-Based Pursuit-Evasion

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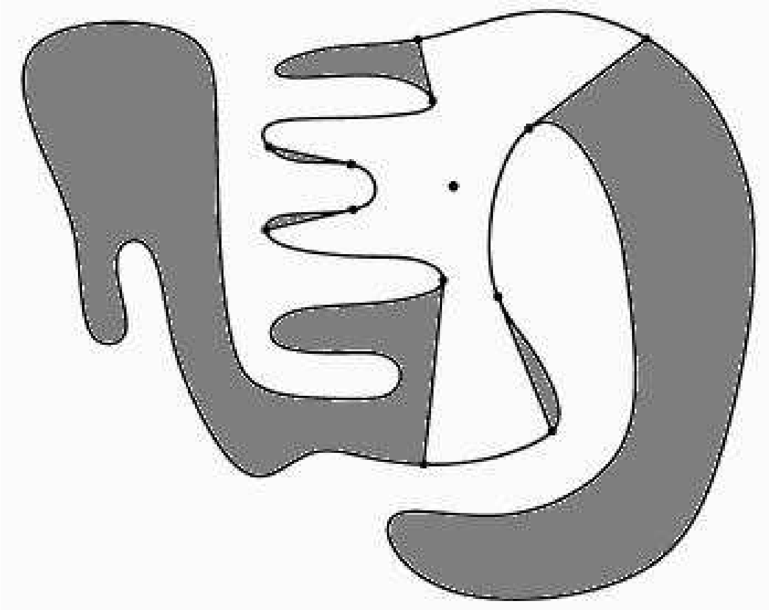
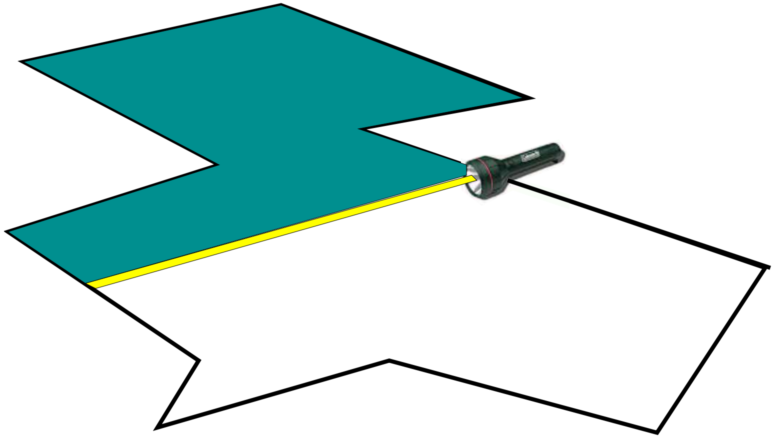
Controlling Wild Bodies

- A 2D environment, possibly curved
- Unpredictable point “evaders” move with unbounded speed
- Point “pursuers” use visibility sensors to find all evaders





Recall: Shadow Information Spaces

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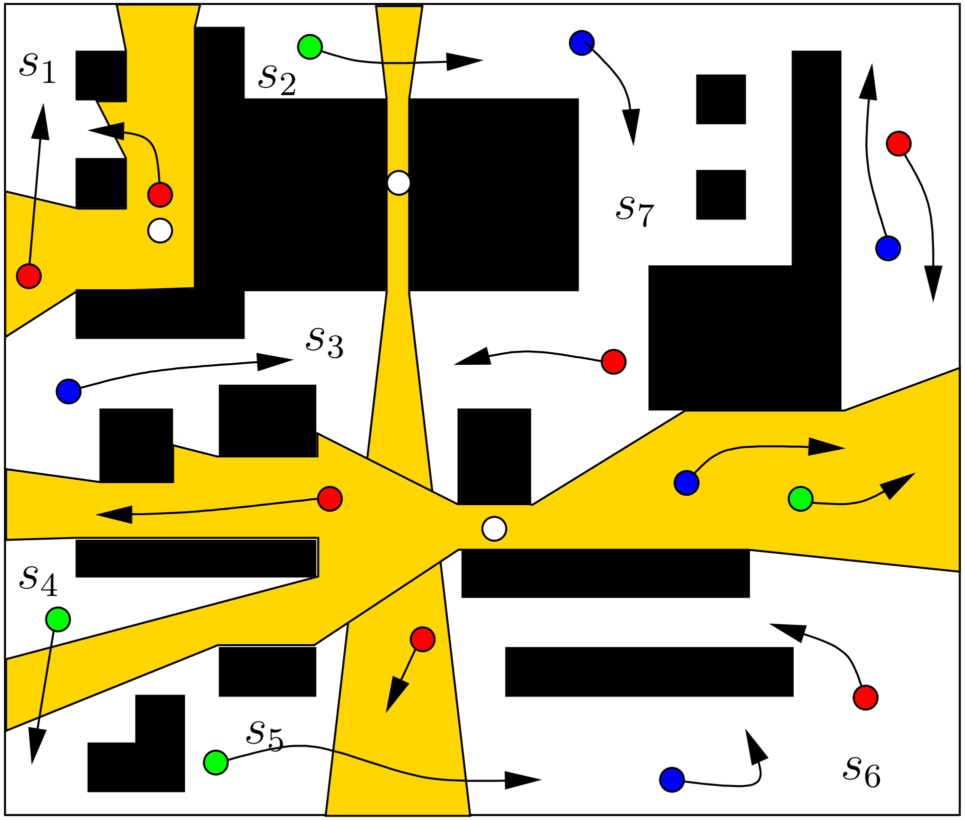

Keep track of bodies out of view–in the shadows.

How many are there? What kinds of bodies are there?

From Filtering to Actuation

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- Key exploited property in filters: **Motion continuity**

From Filtering to Actuation

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Controlling Wild Bodies

- Key exploited property in filters: **Motion continuity**
- Bring in **actuation**, but continue with minimalism, reduced I-spaces
- Passive $\longrightarrow$ Avoid state estimation

  Active $\longrightarrow$ **Avoid system identification**

- What is the new key property?

From Filtering to Actuation

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- Key exploited property in filters: **Motion continuity**
- Bring in **actuation**, but continue with minimalism, reduced I-spaces
- Passive $\longrightarrow$ Avoid state estimation

  Active $\longrightarrow$ **Avoid system identification**

- What is the new key property? **Wildness**

Our Simple Robot

From filters to planning
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Properties?

From filters to planning
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- No map is given in advance

Properties?

From filters to planning
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Controlling Wild Bodies

- No map is given in advance
- No position estimation is available

Properties?

- No map is given in advance
- No position estimation is available
- No system identification has been performed

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Properties?

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- No map is given in advance
- No position estimation is available
- No system identification has been performed
- No sensors, inside or outside of the robot

Properties?

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Controlling Wild Bodies

- No map is given in advance
- No position estimation is available
- No system identification has been performed
- No sensors, inside or outside of the robot
- No computer or any other digital devices

Properties?

From filters to planning
General issues
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Maze searching
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Controlling Wild Bodies

- No map is given in advance
- No position estimation is available
- No system identification has been performed
- No sensors, inside or outside of the robot
- No computer or any other digital devices
- Only one motor, oscillating at 2Hz

A Weasel Ball

From filters to planning
General issues
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Controlling Wild Bodies

An old, popular toy (costs about $4)

From filters to planning
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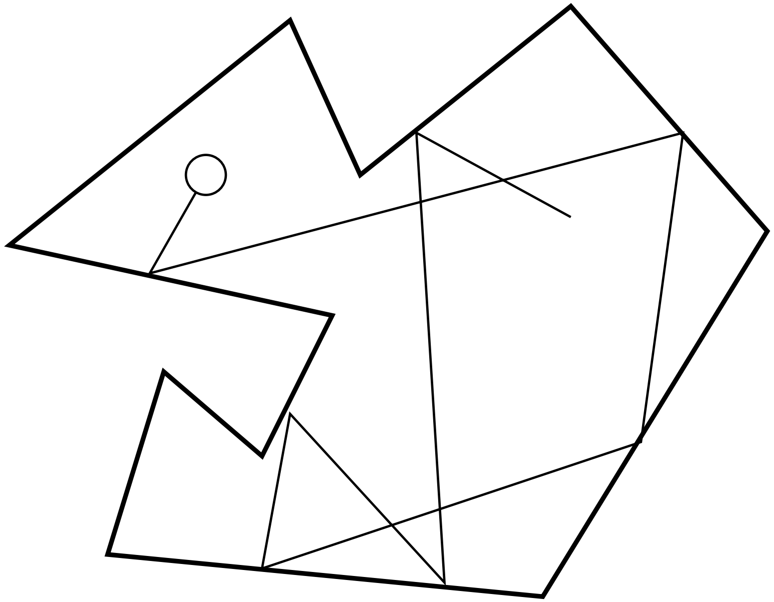
Controlling Wild Bodies

We say that a body is *wild* in a region $R \subseteq \mathbb{R}^2$ if it moves on a trajectory that causes it to repeatedly strike every open interval in ∂R (the boundary of R), with non-zero, non-tangential velocities.



Somewhat informal

Interesting, But How to Control?

From filters to planning
General issues
Visibility-based pursuit evasion
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Gap navigation trees
Learning convex hulls of landmarks
Bug algorithms
Sensorless manipulation
Controlling Wild Bodies

- It is far from stable
- Almost impossible to predict
- Dynamical system modeling or identification is difficult

Interesting, But How to Control?

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- It is far from stable
- Almost impossible to predict
- Dynamical system modeling or identification is difficult

Hmm...the situation is similar for humans.

Manipulating Humans with Gentle Guidance

From filters to planning
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How to clear out the breakfast area after 9:30am?

From filters to planning
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Other Examples

Also: bug traps

Some Related Research

From filters to planning
General issues
Visibility-based pursuit evasion
Maze searching
Gap navigation trees
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Sensorless manipulation
Controlling Wild Bodies

- Tray tilting, Mason, Erdmann, 1988
- Virtual fences for herding cows, Butler, Corke, Peterson, Rus, 2004.
- Manipulation by vibration, Canny, Reznick, 1998; Vose, Lynch, 2011
- Building evacuation, Chalmet, Francis, Saunders, *Fire Technology*, 1982.

From filters to planning
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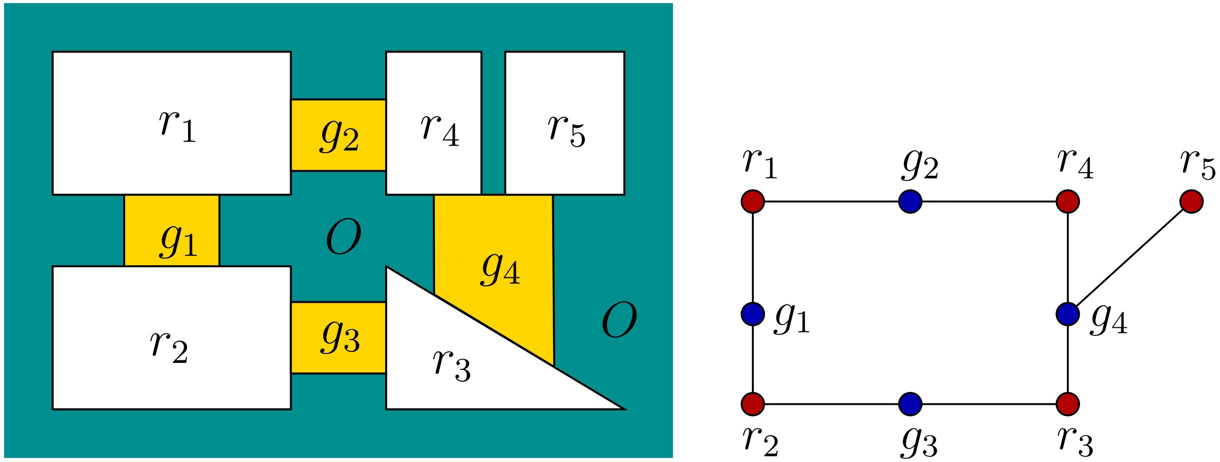
Controlling Wild Bodies



The plane $\mathbb{R}^2$ is partitioned into:
1) *obstacle* set, 2) finite set of *regions*, 3) finite set of *gates*.

A bipartite graph represents the connectivity.

From filters to planning
General issues
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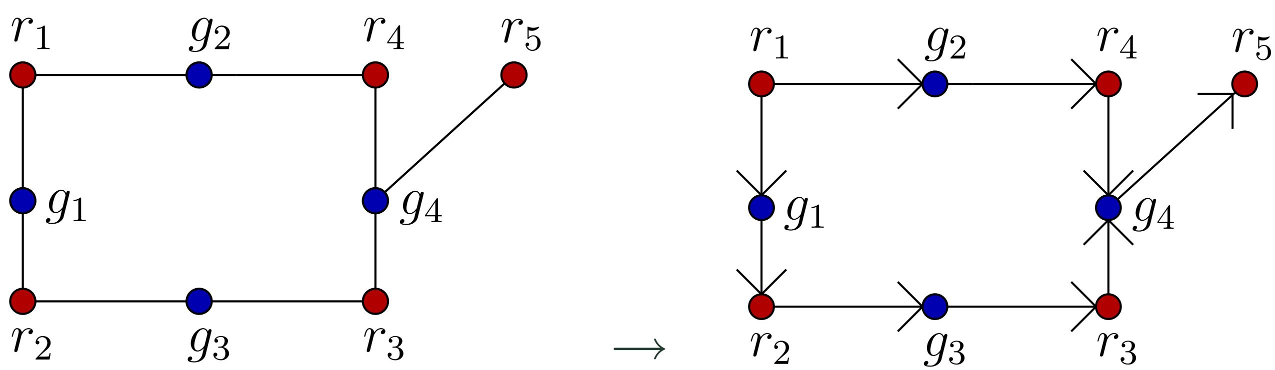
Controlling Wild Bodies

- Design some “wild” bodies
- Place bodies into regions
- Design gates to control them at the region level

Imagine an unusual hybrid system

A discrete flow across regions can be obtained



Types of Gates

From filters to planning
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Controlling Wild Bodies

- **Static gates:** The gates are fixed in advance and allow one-way motions from region to region.

- **Pliant gates:** The gates have internal modes that affect how bodies are permitted to transition between regions and the modes may passively change via contact with bodies.

- **Controllable gates:** Based on information states, the gate modes are externally changed during execution.

- **Virtual gates:** Based on robot sensing, and never represent true physical obstructions.

Engineering a Static Gate

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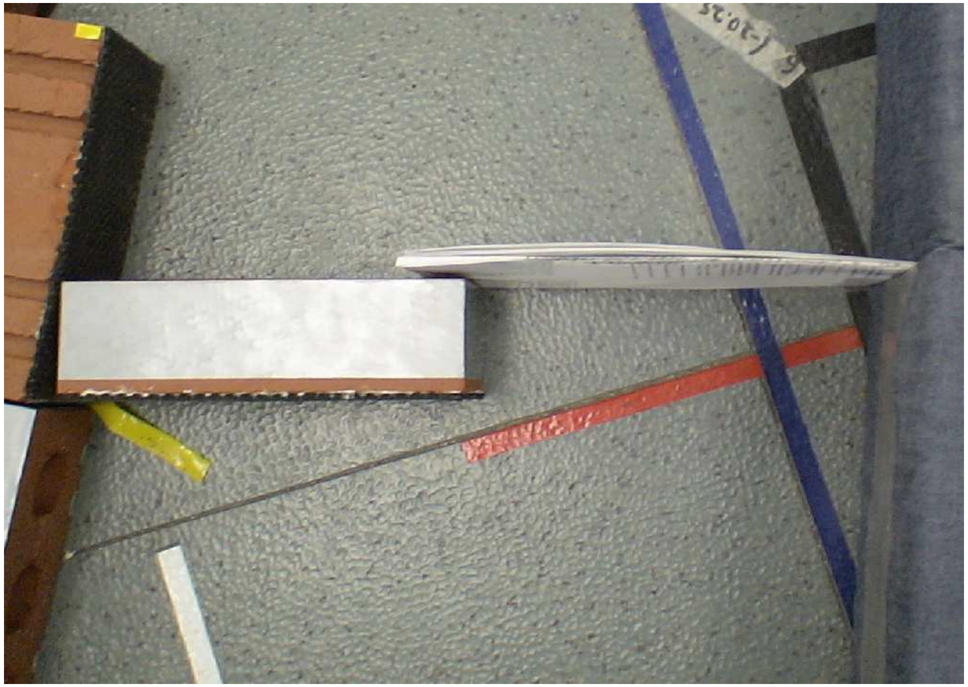

Strips of paper, wedged between bricks

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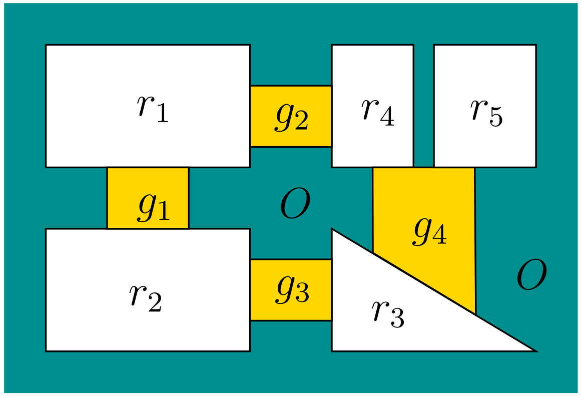

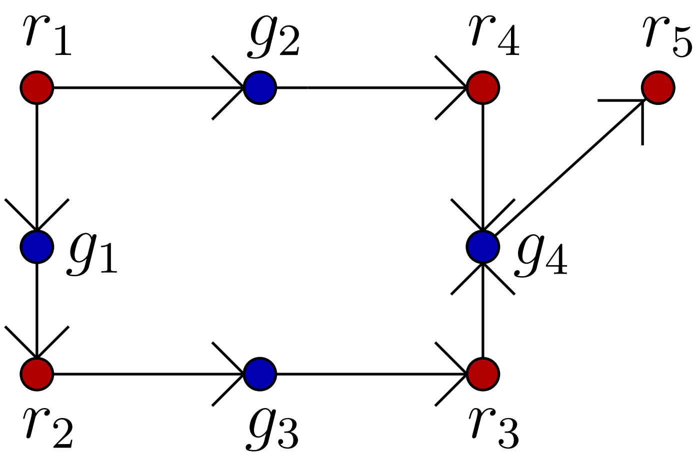

Compute a discrete flow to the goal region (BFS, Dijkstra).

Related work: Sequential composition of funnels, Lozano-Perez, Mason, Taylor, 1984; Mason, Goldberg, 1990; Burridge, Rizzi, Koditschek, 1999; Conner, Rizzi, Choset, 2003.

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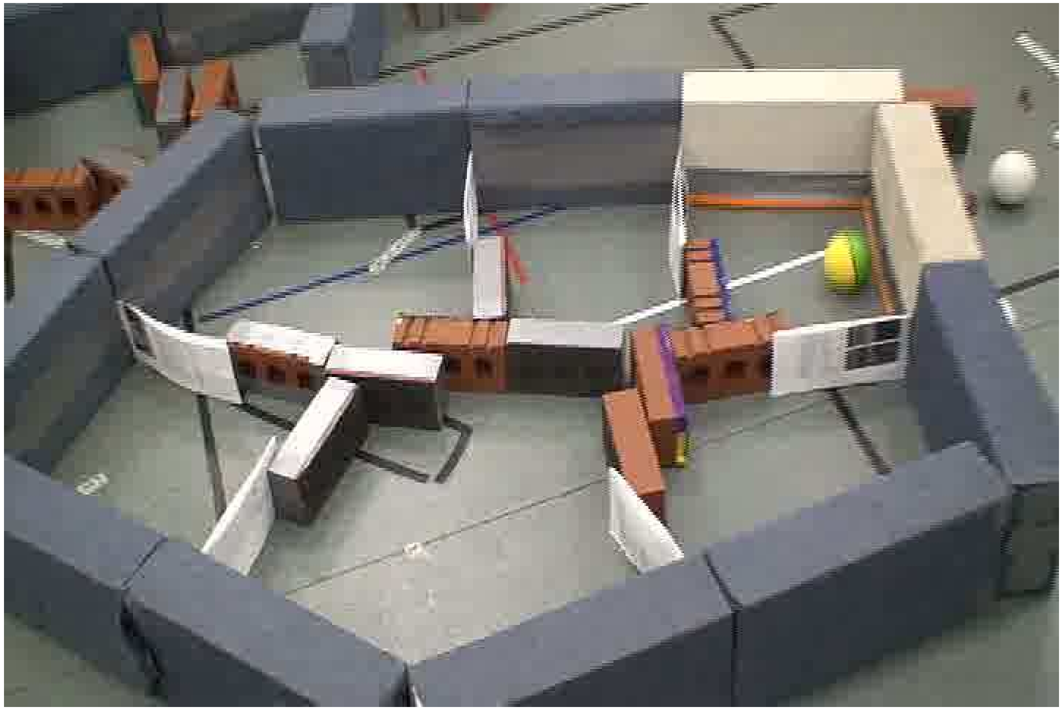

Goal: Flow to lower left region.

Static Gates: Multi-Body Navigation

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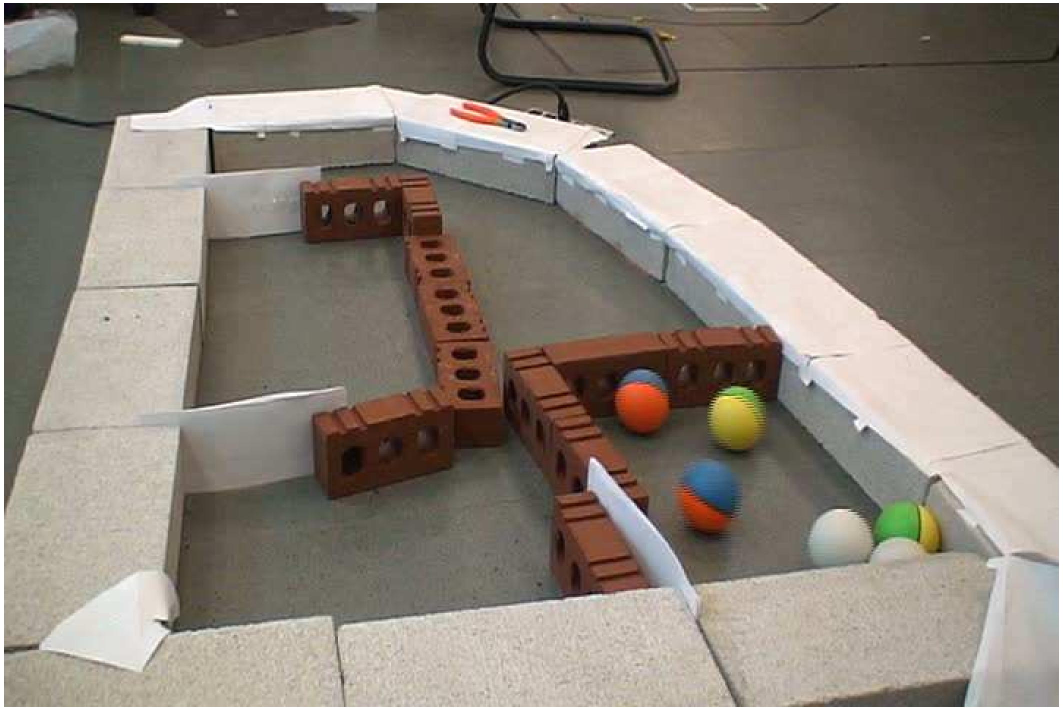

Six balls must flow to the upper right region.

Static Gates: Navigation with Hexbug Nanos

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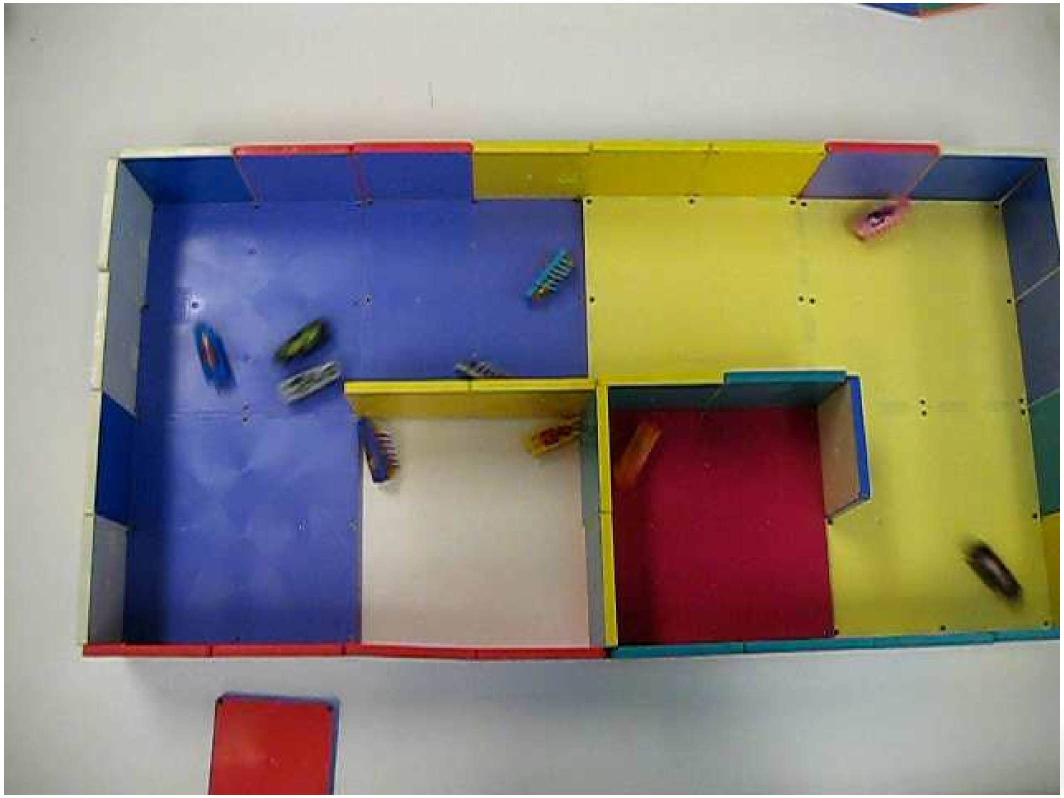

Controlling 10 Hexbug Nanos.

Static Gates: Single-Body Patrolling

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Controlling Wild Bodies

Repeatedly travel a route through all regions.

Static Gates: Multi-Body Patrolling

From filters to planning

General issues

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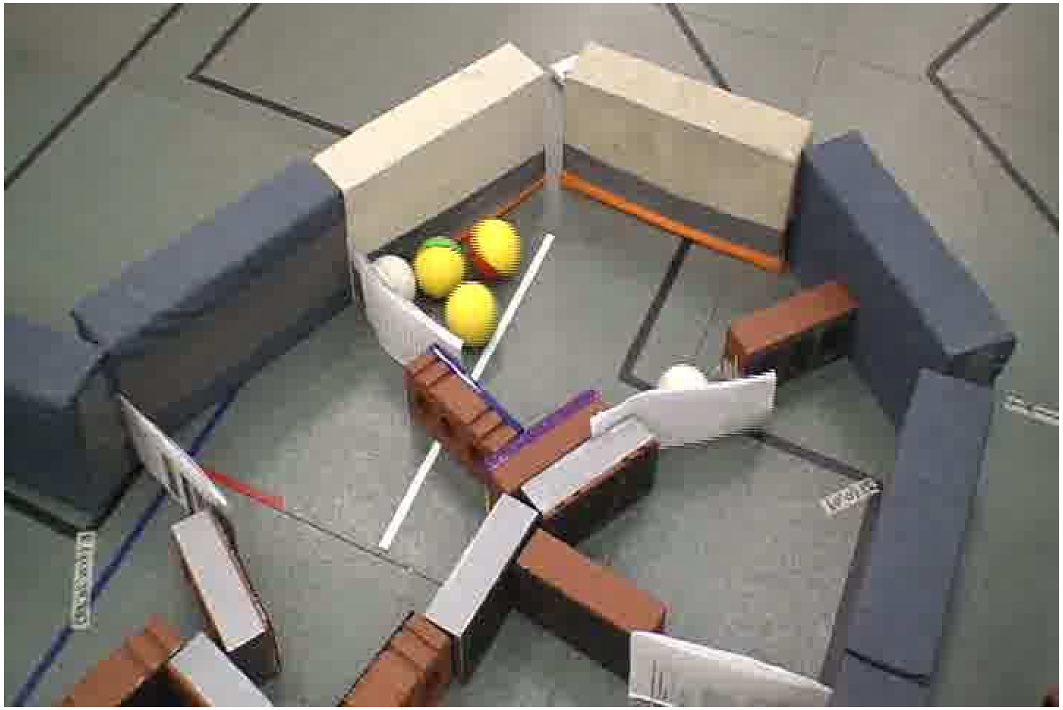
Controlling Wild Bodies



Send all bodies on patrol, asynchronously.

50 Balls

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Controlling Wild Bodies

A tale of 50 weaselballs...

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Pliant Gates

A pliant gate g has a finite set $M(g)$ of *modes*.

A body coming from region r into a gate g in mode m induces a *mode transition*:

$$m' = f(m, r)$$

Mode transitions are caused by the bodies while they traverse gates.

A Simple Pliant Gate Design

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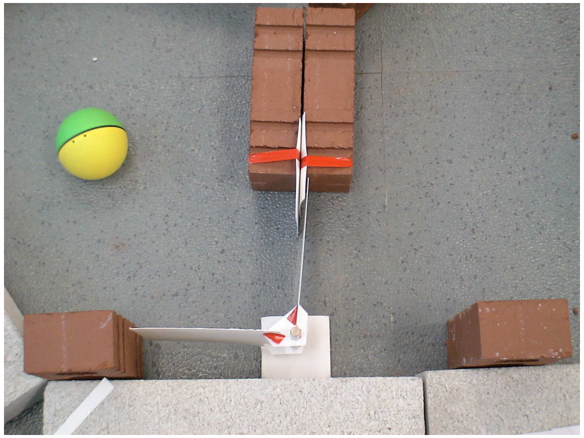
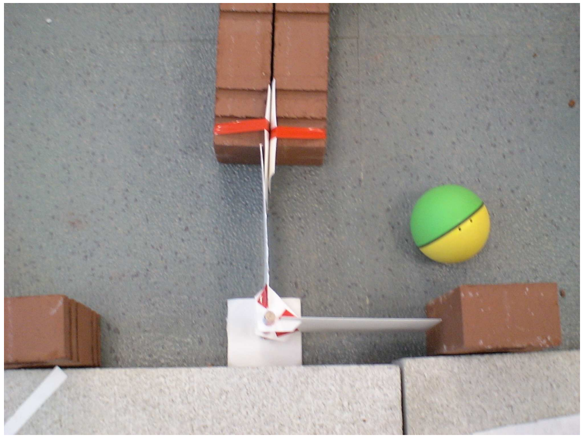


Left to right



Right to left

A two-mode pliant gate that maintains region counts.

Pliant Gates: Two-Way Revolving Door

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Keeping the number of balls roughly constant in each region.

Pliant Gates: Four-Way Revolving Door

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Controllable Gates

From filters to planning
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Now suppose that the mode can be externally set by actuators.
Example modes $M(g)$ per gate g:

1. Block all passage
2. Allow left to right passage only
3. Allow right to left passage only
4. Allow bidirectional passage

Let M be the Cartesian product of all mode sets.

Key issue: What *information* is used to set $m \in M$?

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For some time interval $T = [0, t]$:

$$\pi : T \rightarrow M$$

Time feedback

For sensor with observation space Y:

$$\pi : Y \rightarrow M$$

Sensor feedback

More generally, for any information space $\mathcal{I}$, we have:

$$\pi : \mathcal{I} \rightarrow M$$

Sensor Feedback and Tilting Ramps

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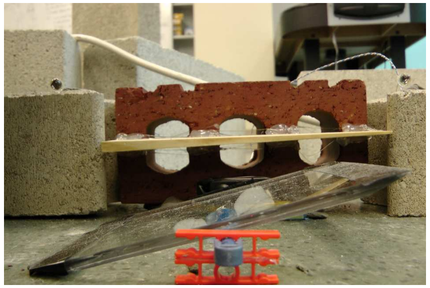
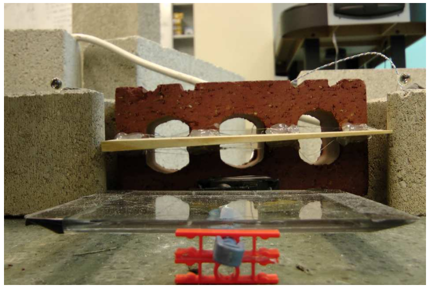
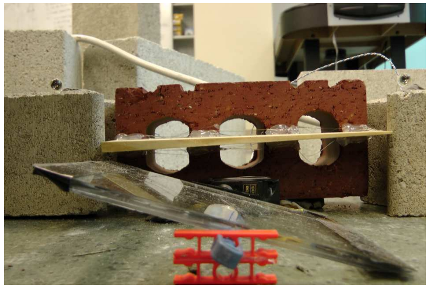
Tilting ramp:



L to R

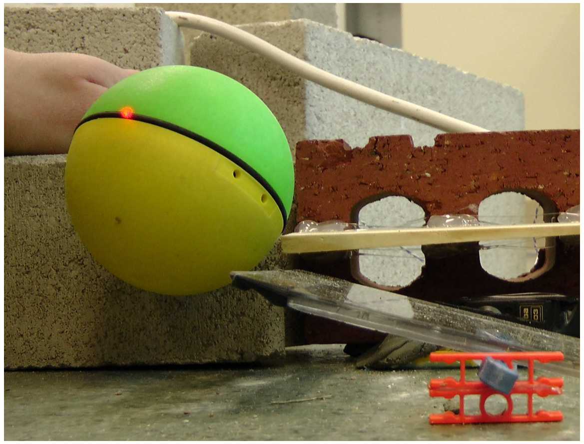


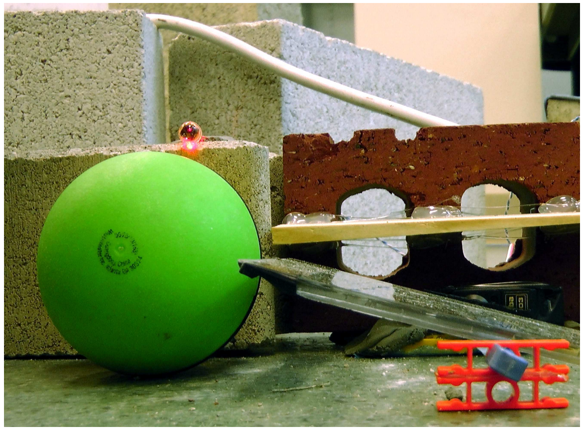
Blocked



R to L

Sensor beam feedback:





What Kinds of Tasks Can We Solve?

From filters to planning
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Visibility-based pursuit evasion
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Consider Linear Temporal Logic (LTL):

- Navigation: $\Diamond \pi_1$
- Sequencing: $\Diamond(\pi_1 \wedge \Diamond(\pi_2 \wedge \Diamond(\pi_3 \wedge \cdots \Diamond \pi_k) \cdots))$
- Coverage: $\Diamond \pi_1 \wedge \Diamond \pi_2 \wedge \cdots \Diamond \pi_k$
- Avoiding regions: $\neg(\pi_1 \vee \pi_2 \cdots \vee \pi_k) \mathcal{U} \pi_{final}$
- Patrolling: $\Box(\Diamond \pi_1 \wedge \Diamond \pi_2 \wedge \ldots \Diamond \pi_k)$.

Examples are from Kress-Gazit, Fainekos, Pappas, 2005.

RSS 2011: From LTL to weaselball implementations.

What Kinds of Tasks Can We Solve?

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Approach:

1. Express the task in some logic
2. Convert into a solution in terms of region sequences
3. Implement using controllable gates and sensor feedback

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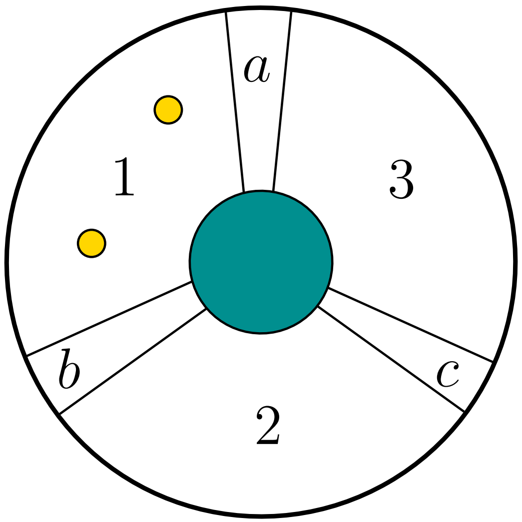

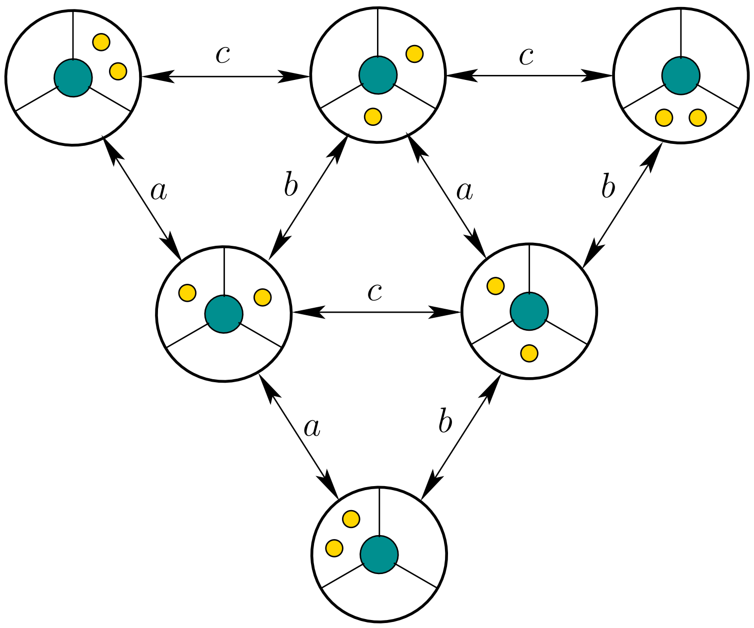

- Imagine indistinguishable balls in boxes.
- There is a natural transition graph.
- Express tasks using logic, and convert to sequences of distributions.

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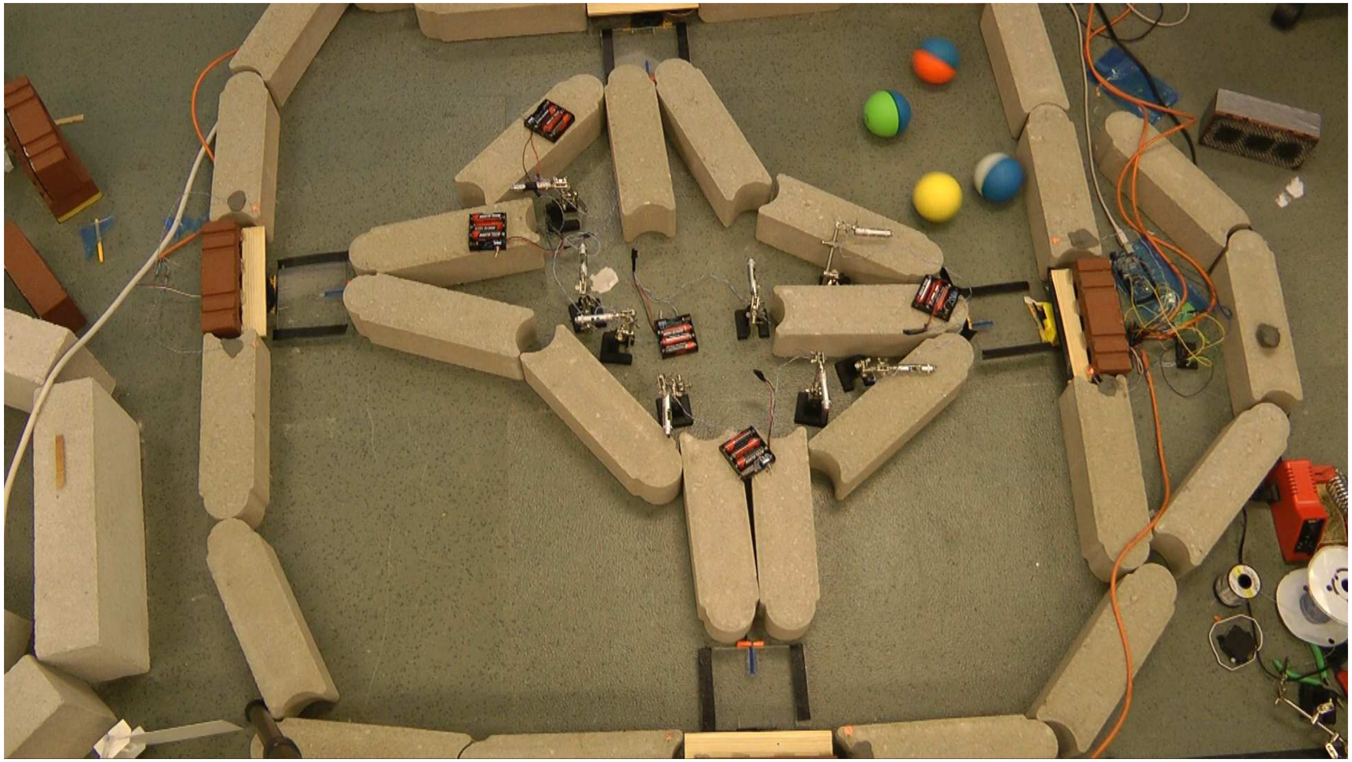
Controlling Wild Bodies

Splitting Video:



Merging Video

Virtual Gates

From filters to planning
General issues
Visibility-based pursuit evasion
Maze searching
Gap navigation trees
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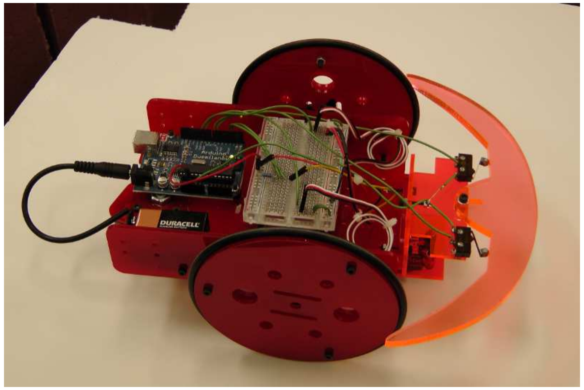
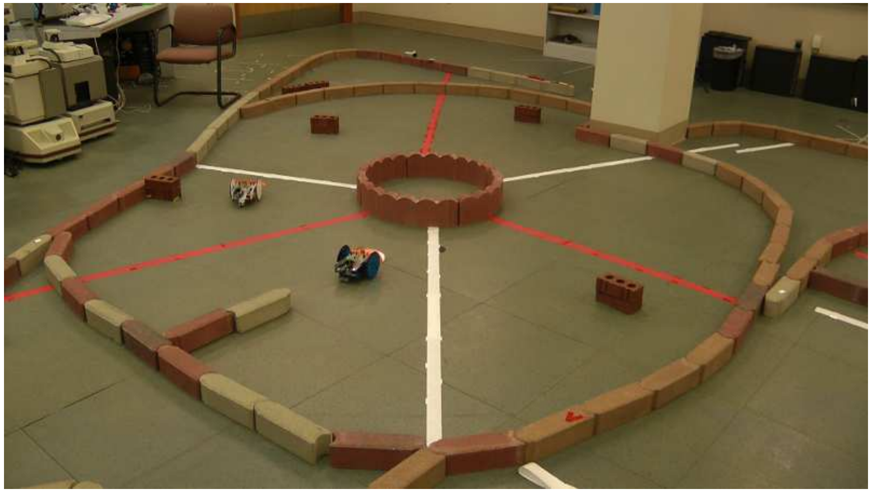


A cheap color sensor detects a virtual gate crossing.



Communication allows simulation of a physical-gate system.

Virtual Gates

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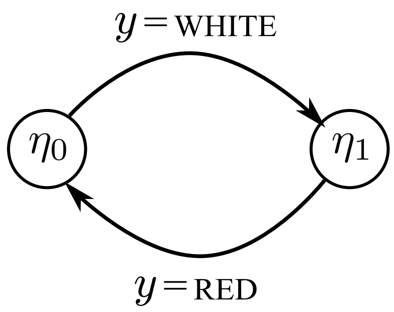
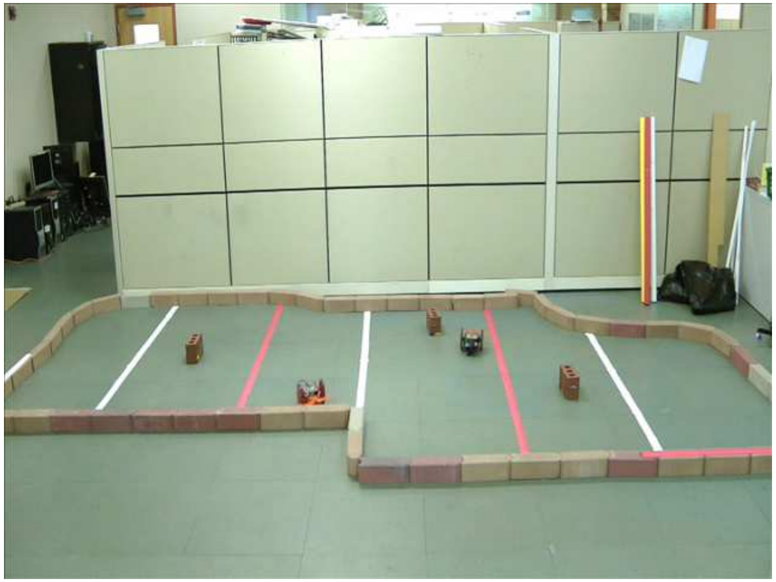
A simple information-feedback plan: $\pi : \mathcal{I} \rightarrow M$

η_0: White open, red closed
η_1: White closed, red open





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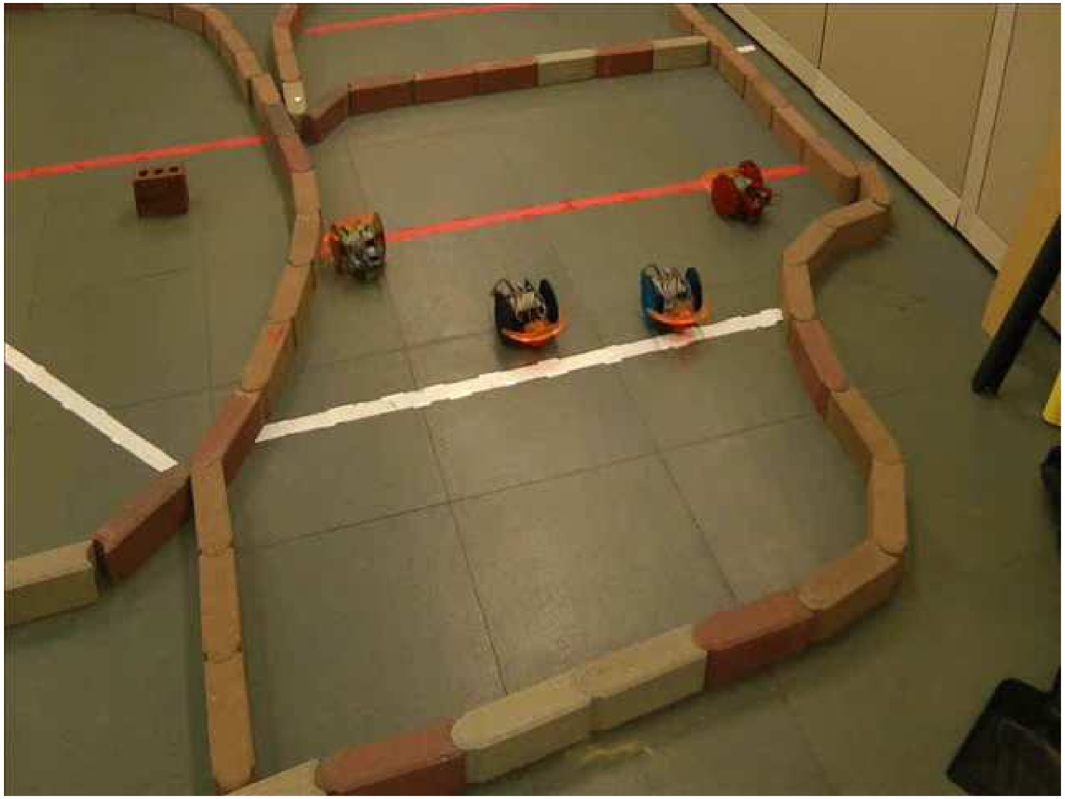
Separation into classes



Each robot can treat the boundaries (red and white) differently.

Information Feedback Using a Combinatorial Filter

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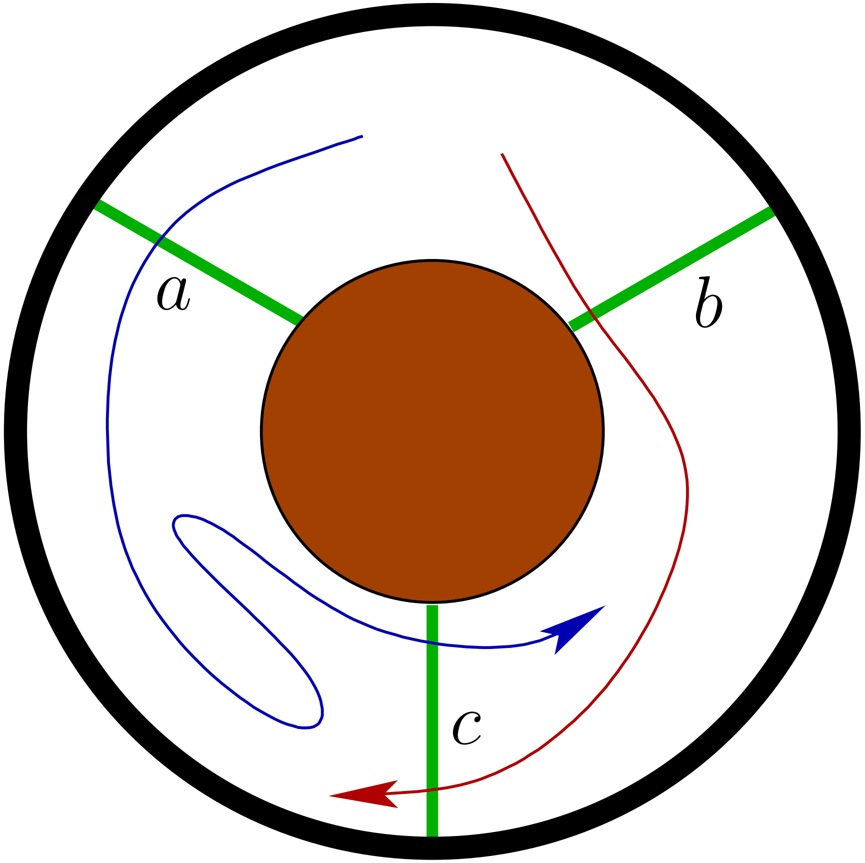

History I-state: $abbacbacabababcabcbba$

Question: Are the bodies in the **same** room?

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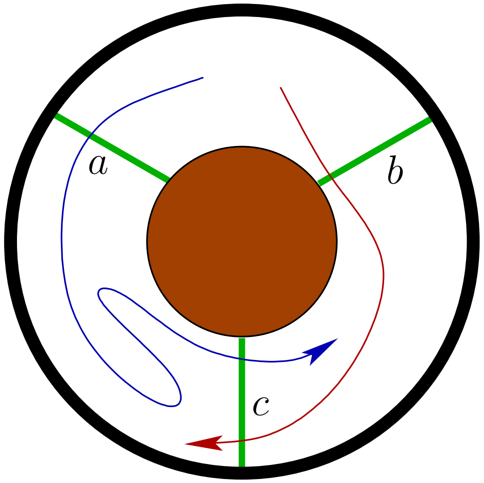
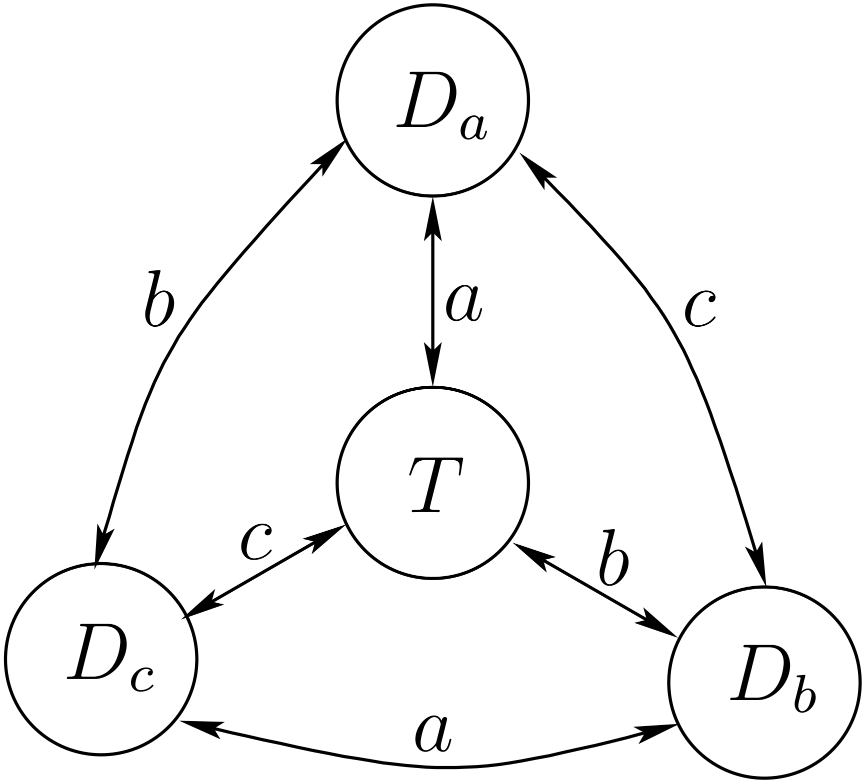
Living in a Tiny Information Space

This two-bit machine can read strings of any length and correctly report the answer.





Three-Room Patrolling

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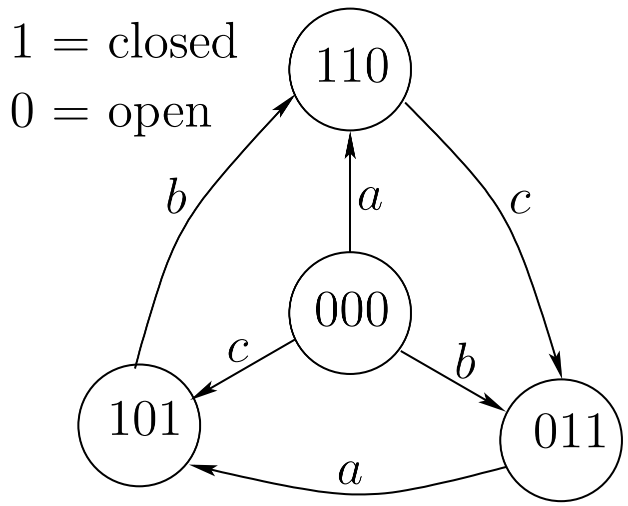

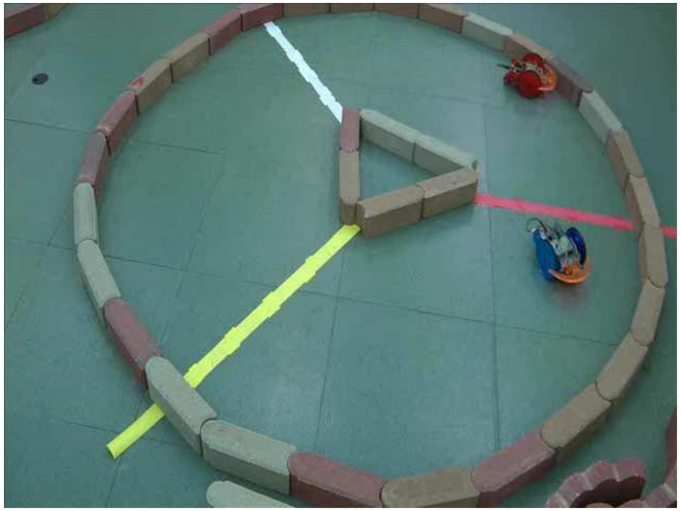

Information space: $\mathcal{I} = \{T, D_a, D_b, D_c\}$

Information feedback plan: $\pi : \mathcal{I} \to M$

Communication is needed between the robots.

Theoretical Design and Analysis

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- Connections to results in mathematics
- Performance analysis
- Designing better motions
- Optimal searching for the gate

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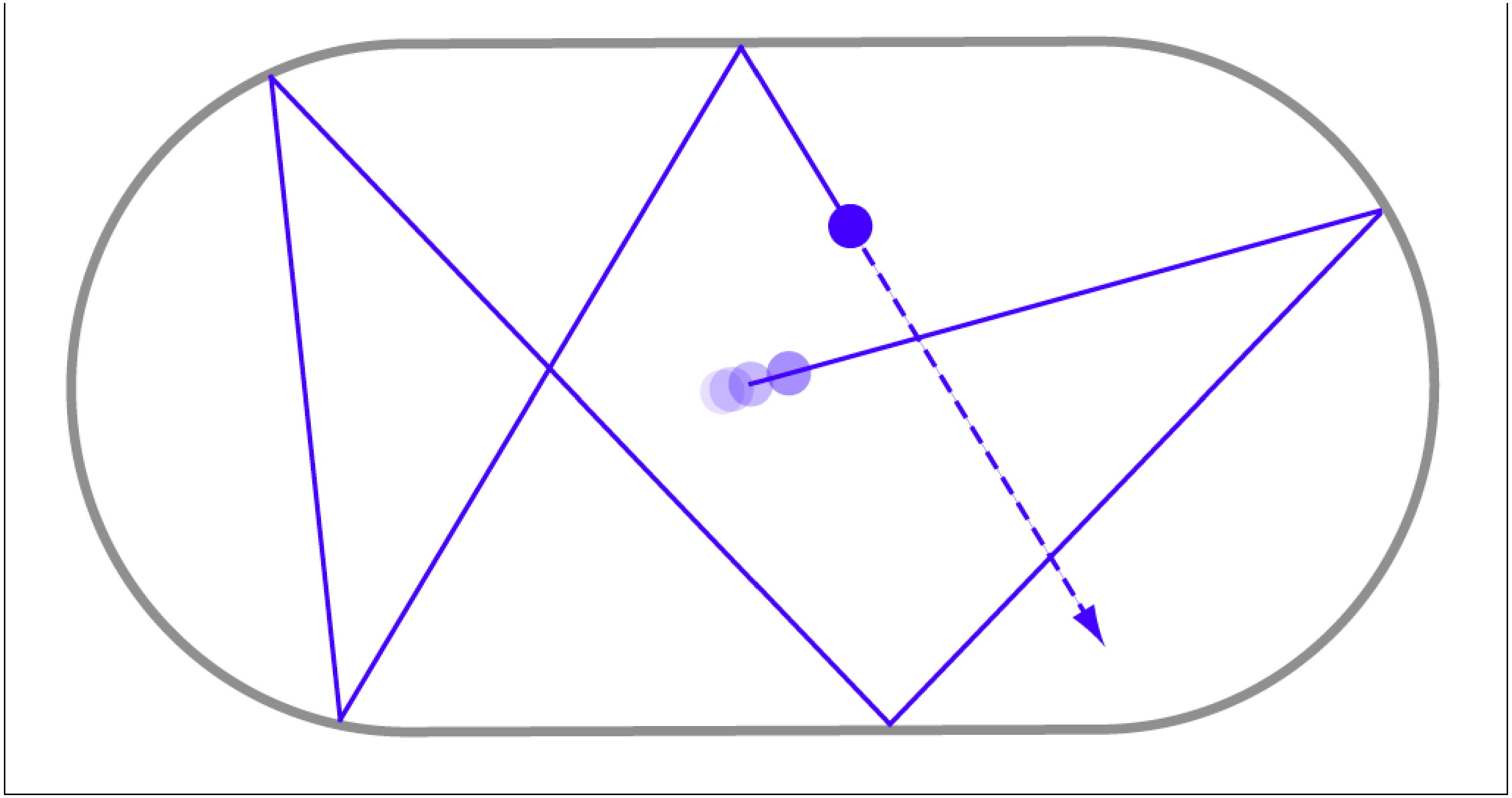
History: Poincare, Hadamard, Artin, Sinai, Bunimovich, ...



Bunimovich stadium

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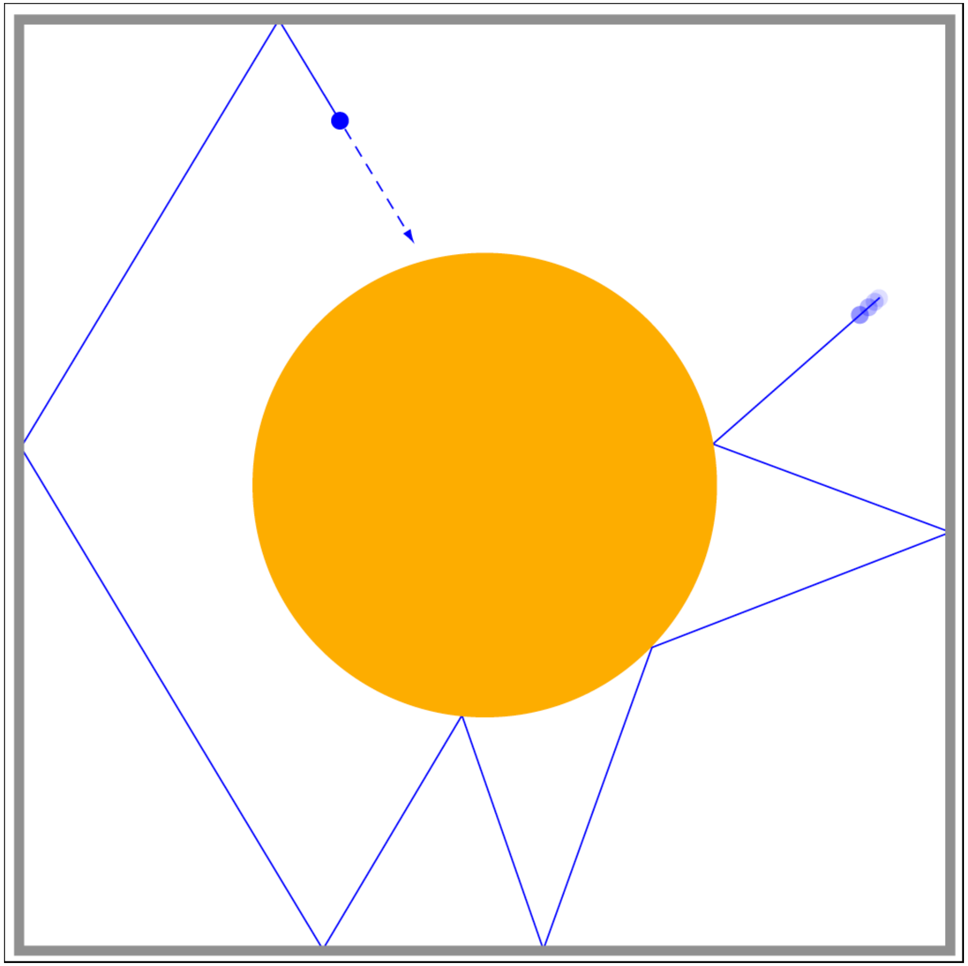

Sinai billiard

Ergodic Dynamics: Definition

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First, a *measure-preserving dynamical system* is a four-tuple $(X, \mathcal{B}, \mu, T)$ for which: 1) X is a set, 2) $\mathcal{B}$ is a σ-algebra over X, 3) $\mu : \mathcal{B} \rightarrow [0, 1]$ is a measure, and 4) $T : X \rightarrow X$ is a measurable transformation that preserves measure (each $A \in \mathcal{B}$ satisfies $\mu(T^{-1}A) = \mu(A)$).

A measurable set $A \in \mathcal{B}$ is called T-invariant mod 0 if $\mu(T^{-1}(A) \bigtriangleup A) = 0$, in which $\bigtriangleup$ denotes the symmetric difference. Note that if this is true then A is T^n-invariant mod 0 for all n.

T is *ergodic* if for every T-invariant mod 0 measurable set A, we have $\mu(A) = 1$ or $\mu(A) = 0$.

Intuition: You can't find a region (connected or not) that traps it.

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Ergodic Dynamics: Intuitive Definition

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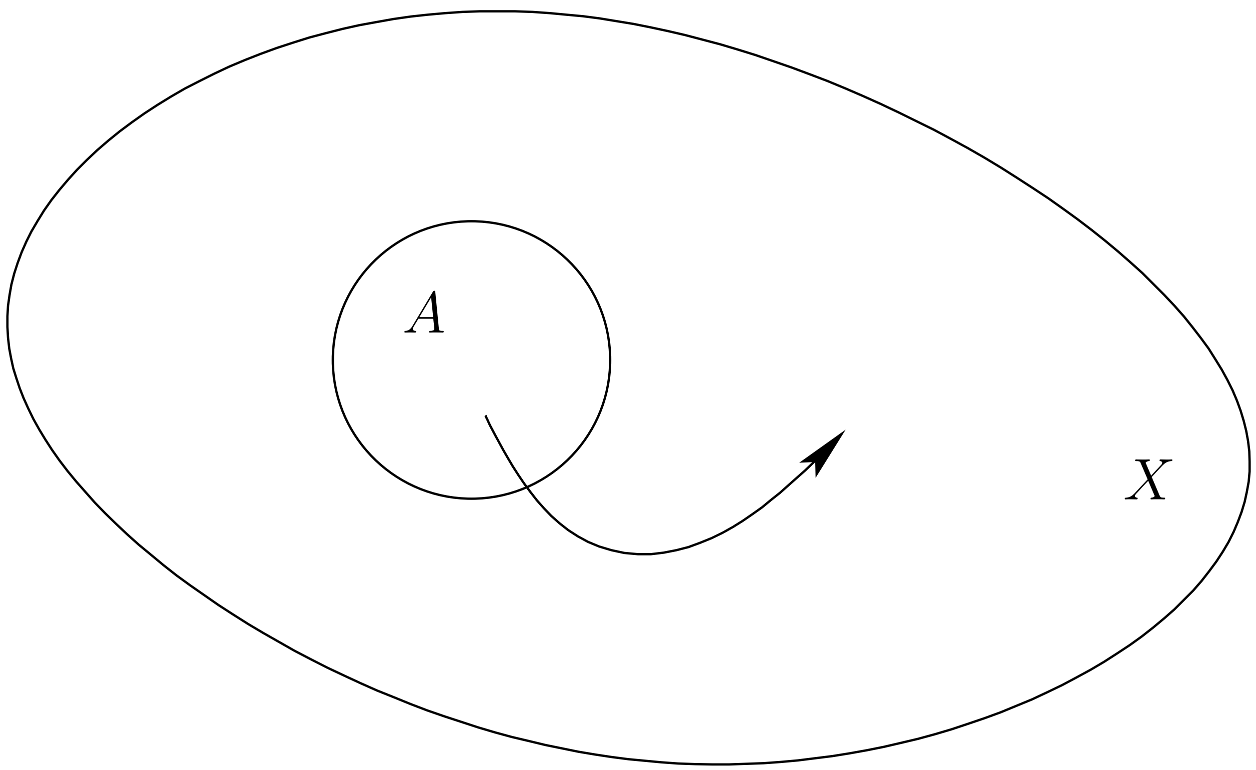
The system is a *measure-preserving transformation* $T : X \to X$ on a state space X.



You can't find a region A (connected or not) that traps the system, unless A or its complement has measure zero.

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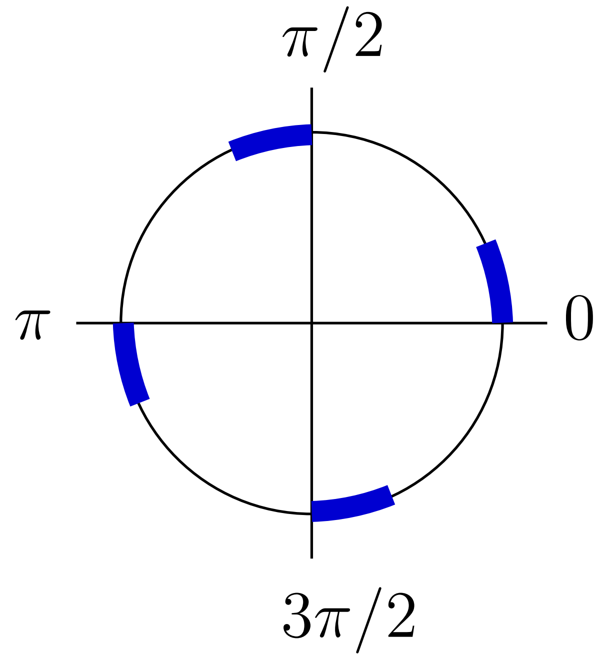
Ergodic Dynamics: Example

Let T be a planar rotation by angle θ.

$X = S^1$

If θ/π is irrational, then T is ergodic; otherwise, it is not.

Example: $\theta = \pi/2$



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Let f by any μ-integrable function.

Time average:

$$\lim_{n \to \infty} \frac{1}{n} \sum_{k=0}^{n-1} f(T^k x)$$

Space average:

$$\frac{1}{\mu(X)} \int f d\mu$$

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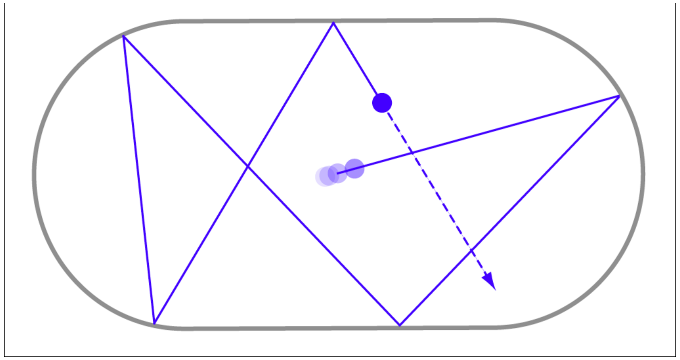
Birkhoff (1931): If T is ergodic, then the time and space averages are the same (almost everywhere).

Example:
Take any $A \subseteq X$
Let $f(x) = 1$ if $x \in A$ and $f(1) = 0$ otherwise.

In this case, Birkoff's theorem states that the frequency of visits to A is equal to $\mu(A)$.



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Ergodicity in Polygons

Kerckhoff, Masur, Smillie, 1986: For almost all polygons and almost all initial conditions, the billiard trajectory is ergodic.

What Is Different About Our Problems?

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- We do not care about *measure-preserving* maps.
- There are many alternative ways to *bounce*.
- Classical ergodicity may be *overkill*.

This is ergodic almost everywhere, but not measure-preserving:

$$f : x \mapsto 2x \mod 1$$

Here, $f : [0,1] \rightarrow [0,1]$

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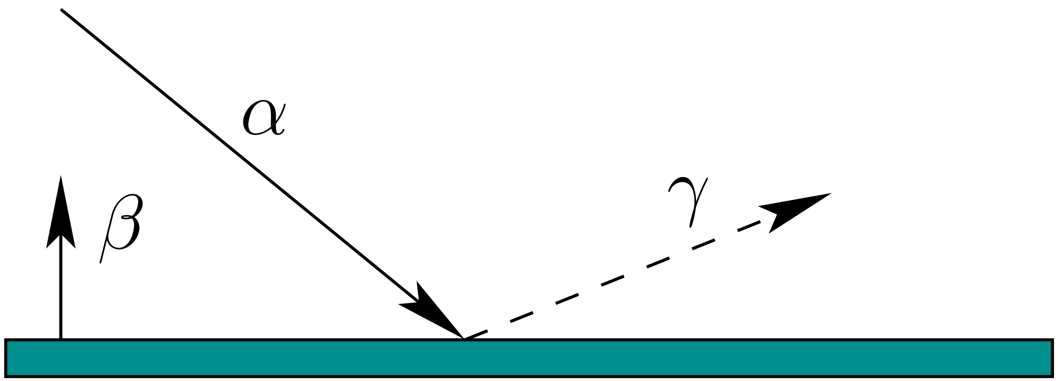

$\gamma = h(\alpha, \beta)$

Fundamental question: What sensors are needed?

Alternative: Select γ randomly (or with $p(\gamma|\alpha, \beta)$)

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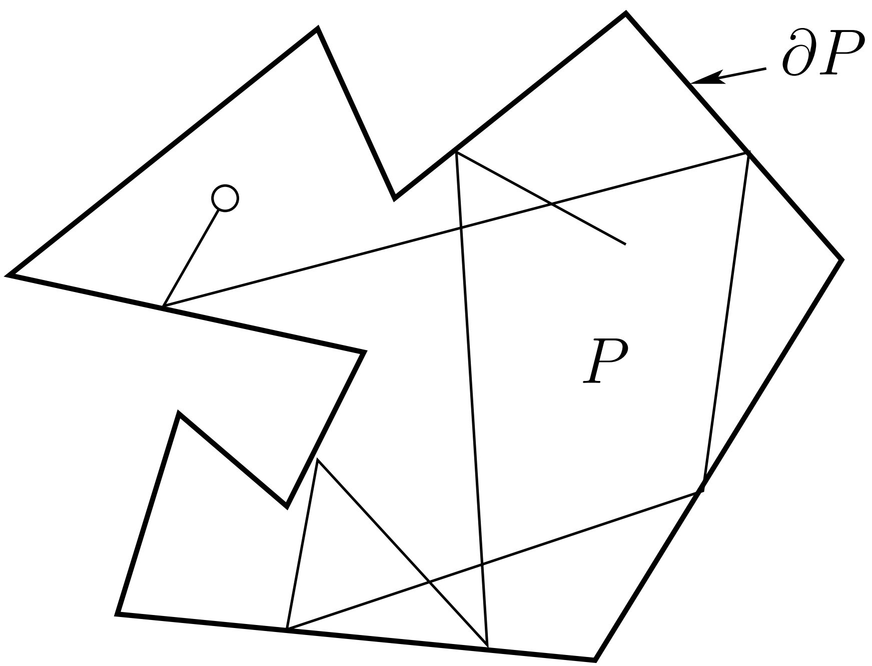
Let $C \subseteq X$. Let $\tilde{x} : [0, \infty) \rightarrow X$ be a trajectory.
$\tilde{x}$ is called *topologically transitive with respect to* C if for every open set $O \subset C$, there exists a time $t > 0$ for which $\tilde{x}(t) \in O$.



Suppose $X \subset \mathbb{R}^3$, in which $(x, y) \in P$ and $\theta \in S^1$.

Possibilities: $C = X$, $C = \partial P \times (0, \pi)$, or $C = \partial P$

Normal Bounce

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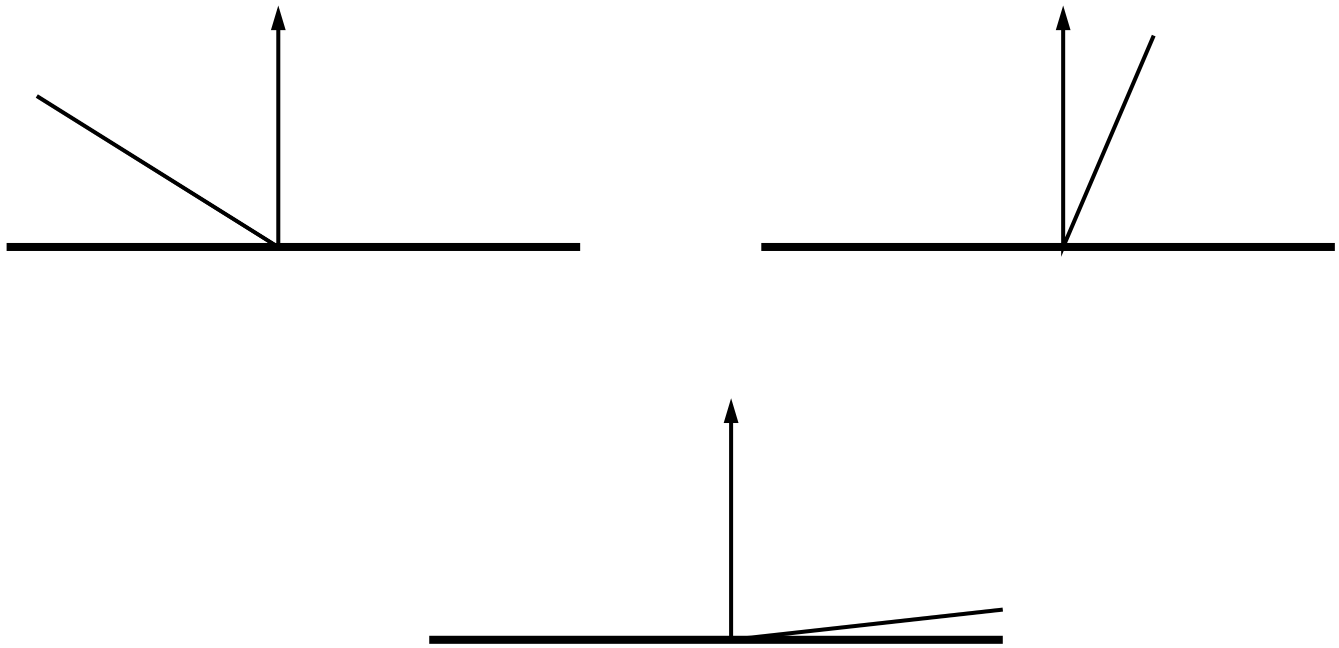
Angle of incoming body not relevant to trajectory after impact.



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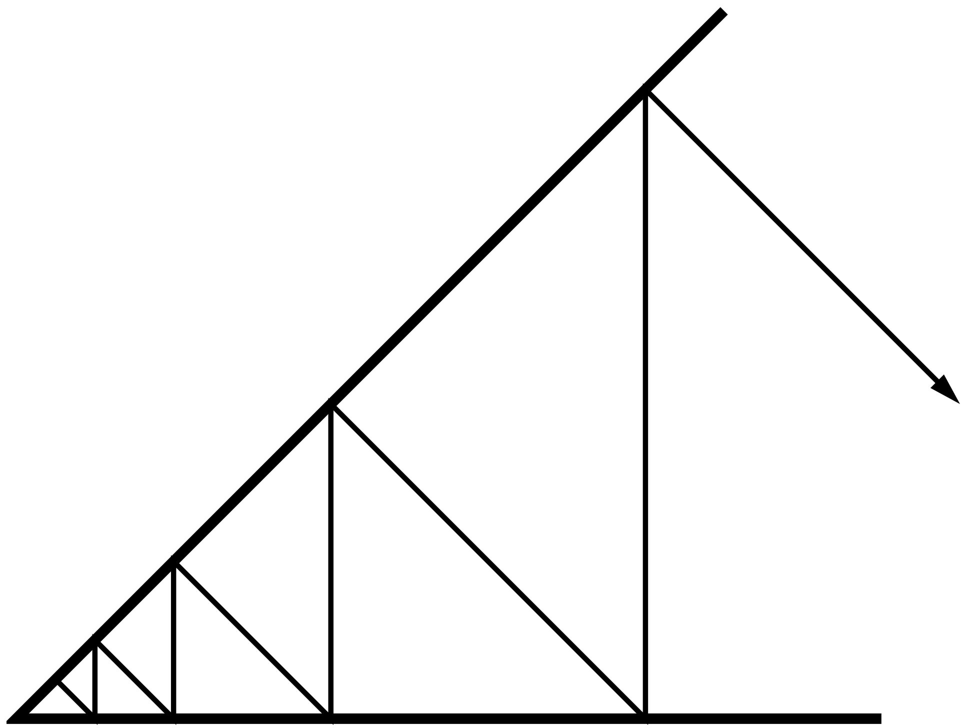
Bodies tend to move away from corners.



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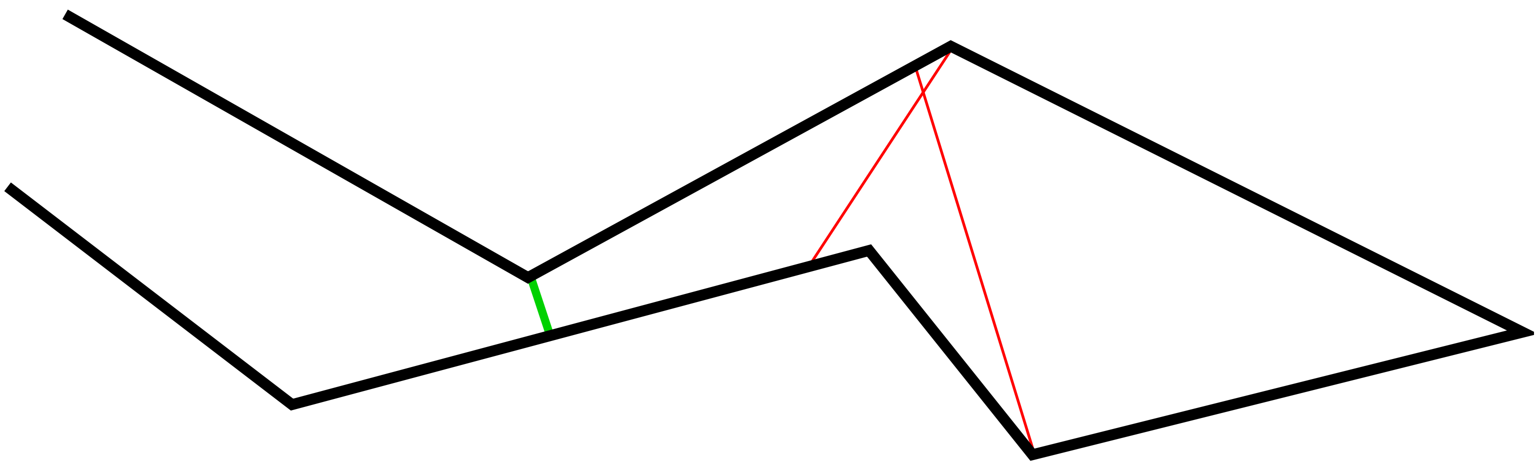
Nothing to the right of the green line will deflect a body back over the green line.



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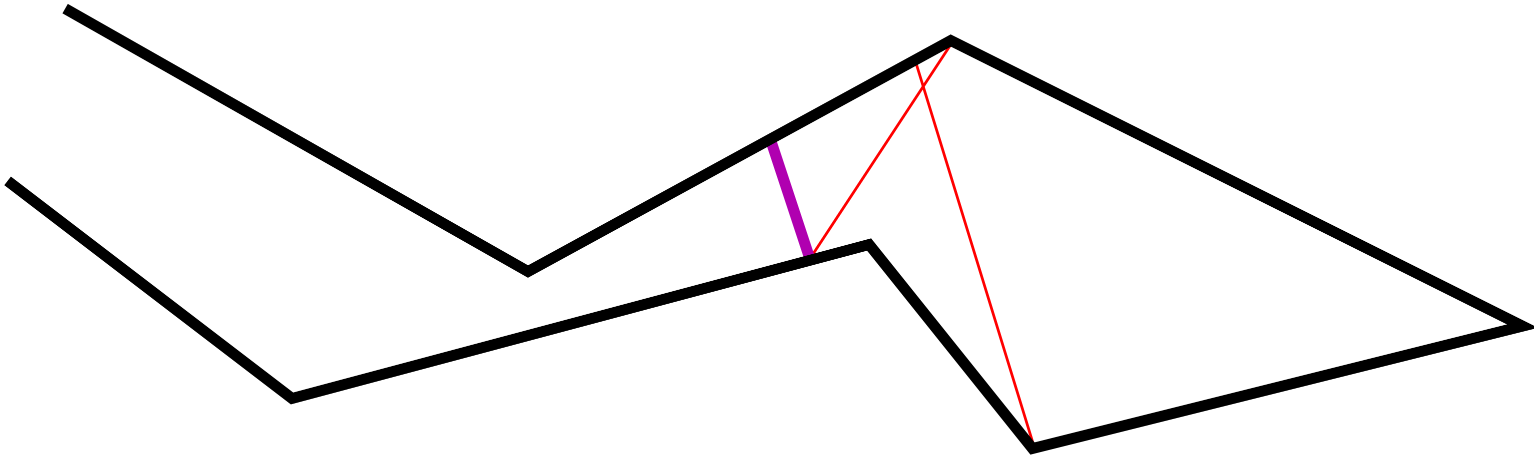
Nothing to the right of the purple line will deflect the body over the green line.



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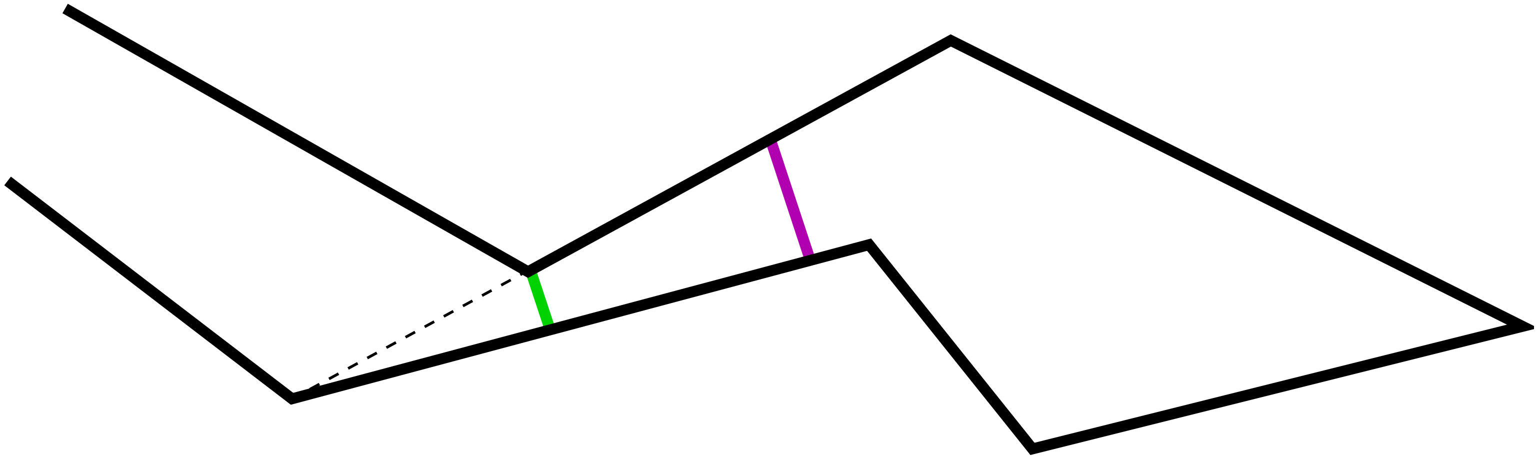
Trap

When the body crosses the green line, it moves toward the purple line, away from the “corner”.



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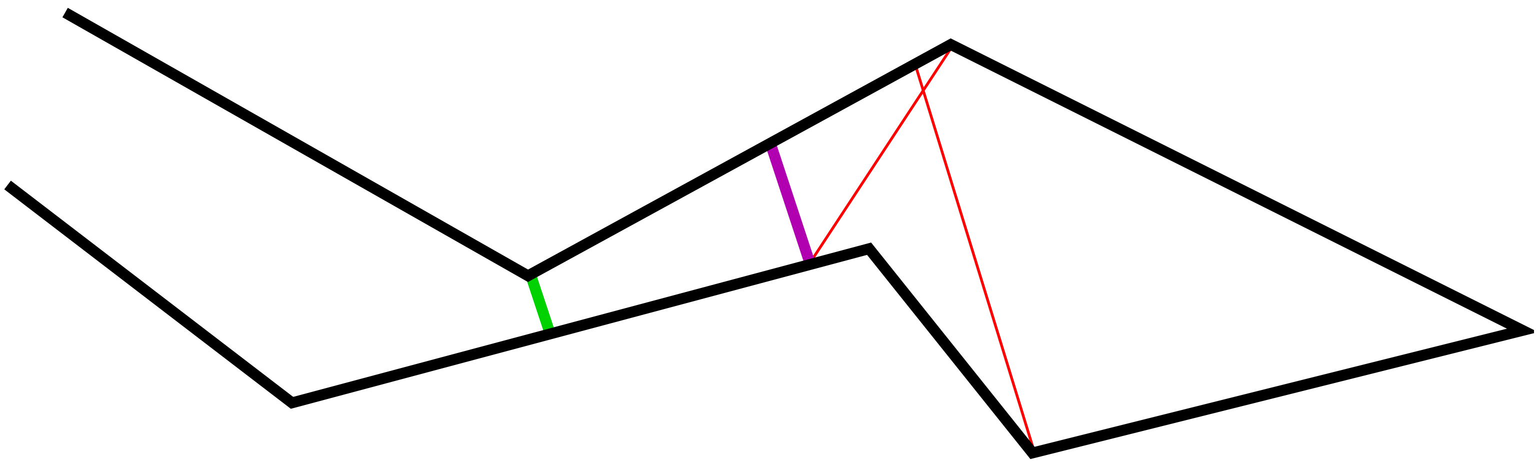
The green and purple lines denote the boundaries of a basin of attraction.



Right Angle Bounce

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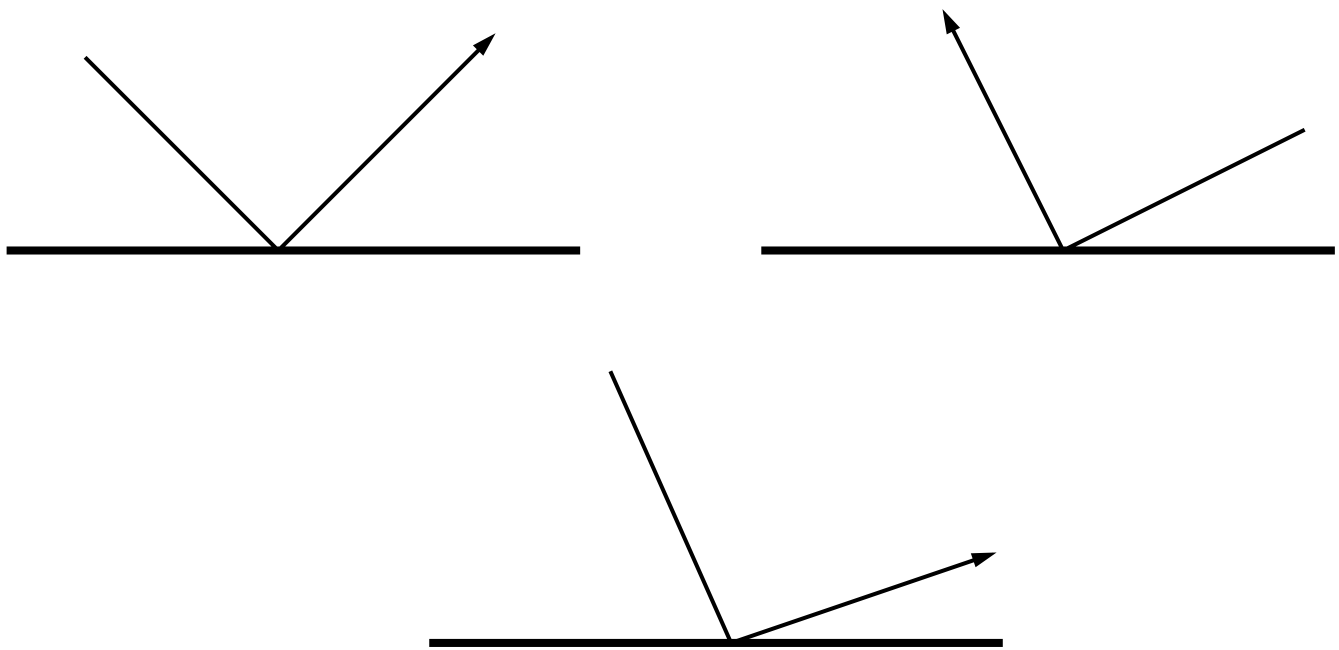
Body always reflects at a right angle, regardless of trajectory relative to wall.



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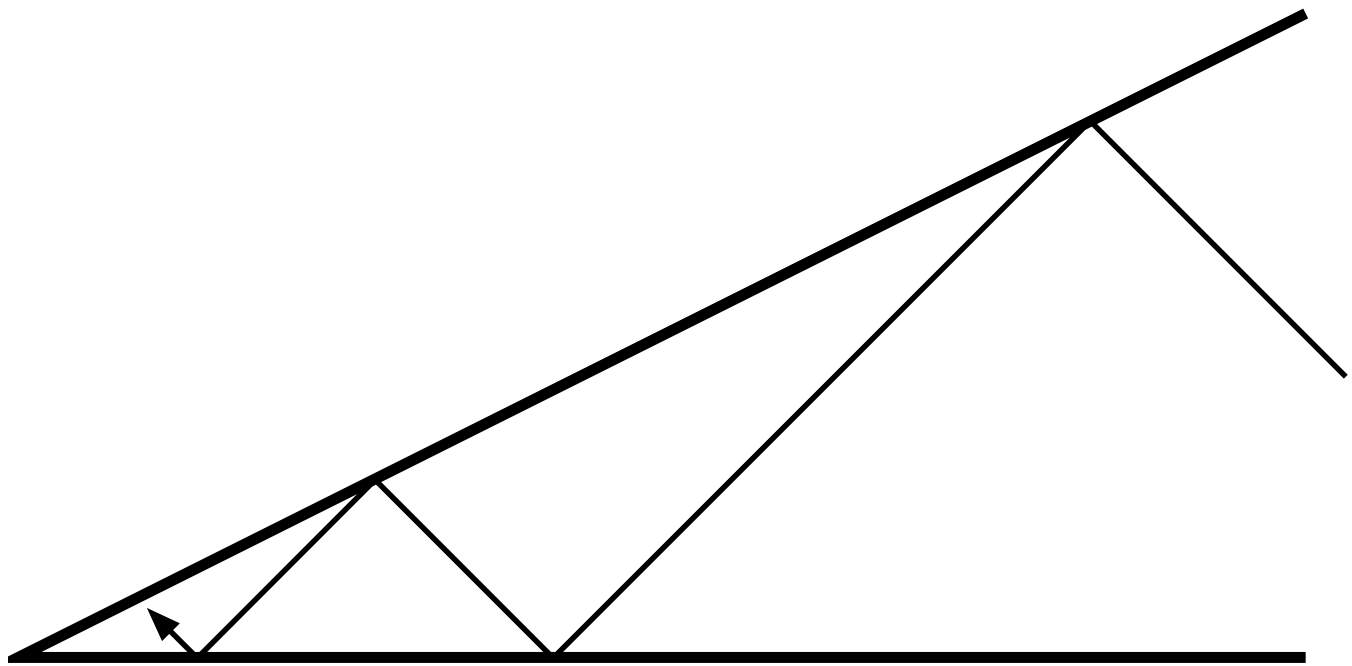
Right angle bouncing is attracted to corners.



Conclusions

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The general paradigm:

- Let the bodies “run wild”, rather than stabilizing.
- Use physical or virtual gates to gently guide them.
- Use as little sensing and comminication as possible.

Challenges:

- Designing more systems of bodies and gates
- Characterizing the space of tasks that can be solved
- Development and analysis of simple bouncing primitives

Part 5 Summary

From filters to planning

General issues

Visibility-based pursuit evasion

Maze searching

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Bug algorithms

Sensorless manipulation

Controlling Wild Bodies

- Use filters to make I-space transitions
- Plan directly in the I-space
- General planning issues
- Need to design virtual sensors, filters, and planning around a task
- Several examples were shown

Although several examples of nice reduced-complexity I-spaces have been found, we have barely scratched the surface...